

# S&P Dow Jones Indices

A Division of **S&P Global**

## Index Mathematics *Methodology*

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# Introduction

This document covers the mathematics of equity index calculations and assumes some acquaintance with mathematical notation and simple operations. The calculations are presented principally as equations, which have largely been excluded from the individual index methodologies, with examples or tables of results to demonstrate the calculations.

## Different Varieties of Indices

A majority of S&P Dow Jones Indices' equity indices are market cap weighted and float-adjusted, where each stock's weight in the index is proportional to its float-adjusted market value.

In addition, S&P Dow Jones Indices offers a variety indices calculated according to various methodologies which are covered in this document:

- Price weighted indices – where constituent weights are determined solely by the prices of the constituent stocks in the index
- Equal weighted indices – where each stock is weighted equally in the index
- Indices weighted by other factors such as maximum weight restrictions or certain attributes used to choose the stocks
- Leveraged and inverse indices – which return positive or negative multiples of their respective underlying indices
- Weighted return indices – commonly known as index of indices, where each underlying index is a component with an assigned weight to calculate the overall index of indices level
- Indices that operate on an index as a whole rather than on the individual stocks – these include calculations of various total return methodologies and index fundamentals
- Dividend indices – which track the total dividend payments of index constituents
- Risk control, excess return, currency, currency hedged, domestic currency return, and the special opening quotation calculations

## The Index Divisor

Throughout all the calculations there is one concept that is crucially important to understanding how indices are calculated – the index divisor.

The simplest capitalization weighted index can be thought of as a portfolio consisting of all available shares of the stocks in the index. While one might track this portfolio's value in dollar terms, it would probably be an unwieldy number – for example, the S&P 500 float-adjusted market value is a figure in the trillions of dollars. Rather than deal with ten or more digits, the figure is scaled to a more easily handled number (e.g. 2000). Dividing the portfolio market value by a factor, usually called the divisor, does the scaling.

An index is not exactly the same as a portfolio. For instance, when a stock is added to or deleted from an index, the index level should not jump up or drop down; while a portfolio's value would usually change as stocks are swapped in and out. To assure that the index's value, or level, does not change when stocks are added or deleted, the divisor is adjusted to offset the change in market value of the index. Thus, the divisor plays a critical role in the index's ability to provide a continuous measure of market valuation when faced with changes to the stocks included in the index. In a similar manner, some corporate actions that cause changes in the market value of the stocks in an index should not be reflected in the index level. Adjustments are made to the divisor to eliminate the impact of these corporate actions.

# Capitalization Weighted Indices

Many of S&P Dow Jones Indices' equity indices, indeed the most widely quoted stock indices, are capitalization-weighted indices. Sometimes these are called value-weighted or market cap weighted instead of capitalization weighted. Examples include the S&P 500, the S&P Global 1200 and the S&P BMI indices.

In the discussion below most of the examples refer to the S&P 500 but apply equally to a long list of S&P Dow Jones Indices' cap-weighted indices.

## Definition

The formula to calculate the S&P 500 is:

$$Index\ Level = \frac{\sum_i P_i * Q_i}{Divisor} \quad (1)$$

The numerator on the right hand side is the price of each stock in the index multiplied by the number of shares used in the index calculation. This is summed across all the stocks in the index. The denominator is the divisor. If the sum in the numerator is US\$ 20 trillion and the divisor is US\$ 10 billion, the index level would be 2000.

This index formula is sometimes called a “base-weighted aggregative” method.<sup>1</sup> The formula is created by a modification of a *LasPeyres* index, which uses base period quantities (share counts) to calculate the price change. A *LasPeyres* index would be:

$$Index = \frac{\sum_i P_{i,1} * Q_{i,0}}{\sum_i P_{i,0} * Q_{i,0}} \quad (2)$$

In the modification to (2), the quantity measure in the numerator,  $Q_0$ , is replaced by  $Q_1$ , so the numerator becomes a measure of the current market value, and the product in the denominator is replaced by the divisor which both represents the initial market value and sets the base value for the index. The result of these modifications is equation (1) above.

## Adjustments to Share Counts

S&P Dow Jones Indices' market cap-weighted indices are float adjusted – the number of shares outstanding is reduced to exclude closely held shares from the index calculation because such shares are not available to investors. S&P Dow Jones Indices' rules for float adjustment are described in more detail in *S&P Dow Jones Indices' Float Adjustment Methodology* or in some of the individual index methodology documents. As discussed there, for each stock S&P Dow Jones Indices calculates an Investable Weight Factor (IWF) which is the percentage of total shares outstanding that are included in the index calculation.

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<sup>1</sup> This term is used in one of the earlier and more complete descriptions of S&P Dow Jones Indices' index calculations in Alfred Cowles, *Common Stock Indices*, Principia Press for the Cowles Commission of Research in Economics, 1939. The book refers to the “Standard Statistics Company Formula;” S&P was formed by the merger of Standard Statistics Corporation and Poor's Publishing in 1941.

When the index is calculated using equation (1), the variable  $Q_i$  is replaced by the product of outstanding shares and the IWF:

$$Q_i = IWF_i * Total\ Shares_i \quad (3a)$$

At times there are other adjustments made to the share count to reflect foreign ownership restrictions or to adjust the weight of a stock in an index. These are combined into a single multiplier in place of the IWF in equation (3a). In combining restrictions it is important to avoid unwanted double counting. Let FA represent the fraction of shares eliminated due to float adjustment, FR represent the fraction of shares excluded for foreign ownership restrictions and IS represent the fraction of total shares to be excluded based on the combination of FA and FR.

$$\text{If } FA > FR \text{ then } IS = 1 - FA \quad (4a)$$

$$\text{If } FA < FR \text{ then } IS = 1 - FR \quad (4b)$$

and equation (3a) can be written as:

$$Q_i = IS_i * Total\ Shares_i \quad (3b)$$

Note that any time the share count or the IWF is changed, it will be necessary to adjust the index divisor to keep the level of the index unchanged.

### Divisor Adjustments

The key to index maintenance is the adjustment of the divisor. Index maintenance – reflecting changes in shares outstanding, corporate actions, addition or deletion of stocks to the index – should not change the level of the index. If the S&P 500 closes at 2000 and one stock is replaced by another, after the market close, the index should open at 2000 the next morning if all of the opening prices are the same as the previous day's closing prices. This is accomplished with an adjustment to the divisor.

Any change to the stocks in the index that alters the total market value of the index while holding stock prices constant will require a divisor adjustment. This section explains how the divisor adjustment is made given the change in total market value. The next section discusses what index changes and corporate actions lead to changes in total market value and the divisor.

Equation (1) is expanded to show the stock being removed, stock  $r$ , separately from the stocks that will remain in the index:

$$Index\ Level_{t-1} = \frac{(\sum_i P_i * Q_i) + P_r Q_r}{Divisor_{t-1}} \quad (5)$$

Note that the index level and the divisor are now labeled for the time period  $t-1$  and, to simplify this example, that we are ignoring any possible IWF and adjustments to share counts. After stock  $r$  is replaced with stock  $s$ , the equation will read:

$$Index\ Level_t = \frac{(\sum_i P_i * Q_i) + P_s Q_s}{Divisor_t} \quad (6)$$

In equations (5) and (6)  $t-1$  is the moment right before company  $r$  is removed from and  $s$  is added to the index;  $t$  is the moment right after the event. By design,  $Index\ Level_{t-1}$  is equal to  $Index\ Level_t$ . Combining (5) and (6) and re-arranging, the adjustment to the Divisor can be determined from the index market value before and after the change:

$$\frac{(\sum_i P_i * Q_i) + P_r Q_r}{Divisor_{t-1}} = Index Level = \frac{(\sum_i P_i * Q_i) + P_s Q_s}{Divisor_t} \quad (7)$$

Let the numerator of the left hand fraction be called  $MV_{t-1}$ , for the index market value at  $(t-1)$ , and the numerator of the right hand fraction be called  $MV_t$ , for the index market value at time  $t$ . Now,  $MV_{t-1}$ ,  $MV_t$  and  $Divisor_{t-1}$  are all known quantities. Given these, it is easy to determine the new divisor that will keep the index level constant when stock  $r$  is replaced by stock  $s$ :

$$Divisor_t = (Divisor_{t-1}) * \frac{MV_t}{MV_{t-1}} \quad (8)$$

As discussed below, various index adjustments result in changes to the index market value. When these adjustments occur, the divisor is adjusted as shown in equation (8).

In some implementations, including the computer programs used in S&P Dow Jones Indices' index calculations, the divisor adjustment is calculated in a slightly different, but equivalent, format where the divisor change is calculated by addition rather than multiplication. This alternative format is defined here. Rearranging equation (1) and using the term  $MV$  (market value) to replace the summation gives:

$$Divisor = \frac{MV}{Index Level} \quad (9)$$

When stocks are added to or deleted from an index there is an increase or decrease in the index's market value. This increase or decrease is the market value of the stocks being added less the market value of those stocks deleted; define  $CMV$  as the Change in Market Value. Recalling that the index level does not change, the new divisor is defined as:

$$Divisor_{New} = \frac{MV + CMV}{Index Level} \quad (10)$$

or

$$Divisor_{New} = \frac{MV}{IndexLevel} + \frac{CMV}{IndexLevel} \quad (11)$$

However, the first term on the right hand side is simply the Divisor value before the addition or deletion of the stocks. This yields:

$$Divisor_{New} = Divisor_{Old} + \frac{CMV}{IndexLevel} \quad (12)$$

Note that this form is more versatile for computer implementations. With this additive form, the second term ( $CMV/Index Level$ ) can be calculated for each stock or other adjustment independently and then all the adjustments can be combined into one change to the Divisor.

### Necessary Divisor Adjustments

Divisor adjustments are made "after the close" meaning that after the close of trading the closing prices are used to calculate the new divisor based on whatever changes are being made. It is, then, possible to provide two complete descriptions of the index – one as it existed at the close of trading and one as it will exist at the next opening of trading. If the same stock prices are used to calculate the index level for these two descriptions, the index levels are the same.

With prices constant, any change that changes the total market value included in the index will require a divisor change. For cataloging changes, it is useful to separate changes caused by the management of the index from those stemming from corporate actions of the constituent companies. Among those changes driven by index management are adding or deleting companies, adjusting share counts and changes to IWFs and other factors affecting share counts.

**Index Management Related Changes.** When a company is added to or deleted from the index, the net change in the market value of the index is calculated and this is used to calculate the new divisor. The market values of stocks being added or deleted are based on the prices, shares outstanding, IWFs and any other share count adjustments. Specifically, if a company being added has a total market cap of US\$ 1 billion, an IWF of 85% and, therefore, a float adjusted market cap of US\$ 850 million, the market value for the added company used is US\$ 850 million. The calculations would be based on either equation (8) or equation (12) above.

For most S&P Dow Jones Indices equity indices, IWFs and share counts updates are applied throughout the year based on rules defined in the methodology. Typically small changes in shares outstanding are reflected in indices once a quarter to avoid excessive changes to an index. The revisions to the divisor resulting from these are calculated and a new divisor is determined. Equation (12) shows how the impact of a series of share count changes can be combined to determine the new divisor.

**Corporate Action Related Changes.** There are a large range of different corporate actions ranging from routine share issuances or buy backs to less frequent events like spin-offs or mergers. These are listed on the table below with notes about the necessary changes and whether the divisor is adjusted.

Corporate Action	Comments	Divisor Adjustment?
Company added/deleted	Net change in market value determines the divisor adjustment.	Yes
Change in shares outstanding	Any combination of secondary issuance, share repurchase or buy back – share counts revised to reflect change.	Yes
Stock split	Share count revised to reflect new count. Divisor adjustment is not required since the share count and price changes are offsetting.	No
Spin-off	The spin-off is added to the index on the ex-date at a price of zero. Each individual index methodology will specify whether the spin-off will be eligible to remain in the index beyond the ex-date.	No
Change in IWF	Increasing (decreasing) the IWF increases (decreases) the total market value of the index. The divisor change reflects the change in market value caused by the change to an IWF.	Yes
Special dividend	When a company pays a special dividend the share price is assumed to drop by the amount of the dividend; the divisor adjustment reflects this drop in index market value.	Yes
Rights offering	Each shareholder receives the right to buy a proportional number of additional shares at a set (often discounted) price. The calculation assumes that the offering is fully subscribed. Divisor adjustment reflects increase in market cap measured as the shares issued multiplied by the price paid.	Yes

With corporate actions where special dividends such as extraordinary cash or other corporate assets that are distributed to shareholders, the price of the stock will gap down on the ex-date (the first day when a new shareholder is not eligible to receive the distribution). The effect of the divisor adjustment is to prevent this price drop from causing a corresponding drop in the index.

*For more information on the treatment of corporate actions, please refer to S&P Dow Jones Indices' Equity Indices Policies & Practices document.*

## Capped Indices

At times it is desirable to set a maximum weight for some stocks in an index. In some markets, regulations restrict the weight of the largest stock or group of stocks to be less than a certain percentage of a portfolio. This is done by a further adjustment to the share count, beyond the investable weight factor. Since the total weight of all stocks in the index will add up to 100%, reducing the weight of one stock will increase the weight of the others. It is possible that when the largest stock's weight is brought down below some limit, the weight of the next largest – or several next largest – stocks will exceed the limit. Therefore, the process must be iterative. Weights will change over time as stock prices move even if share counts remain constant. If a capped stock enjoys a price run-up, it may exceed the cap. In most cases buffer zones are used (e.g. if the maximum allowable weight is 10% the stock's shares are adjusted downward until its weight is 9% leaving a one-percentage point buffer before another adjustment is necessary). Various forms of capped indices are covered in the following sections of this document.

# Price Weighted Indices

## Definition

In a price weighted index, such as the Dow Jones Industrial Average, constituent weights are determined solely by the prices of the constituent stocks. Shares outstanding are set to a uniform number throughout the index. Indices using this methodology will adjust the index divisor for any price impacting corporate action on one of its member stocks; this includes price adjustments, special dividends, stock splits and rights offerings. The index divisor will also adjust in the event of an addition to or deletion from the index.

All other index calculation details follow the standard divisor based calculation methodology detailed in the previous capitalization weighted section.

For more details on corporate action treatment please refer to the table below.

Corporate Action	Comments	Divisor Adjustment?
Company added/deleted	Net change in price determines the divisor adjustment.	Yes
Change in shares outstanding	Share changes due to secondary issuance, share repurchase, buy back, or any other reason are not recognized in the price weighted methodology. All shares are set to 1.	No
Stock split	Share count is not revised to reflect new count. Divisor adjustment is required to offset the adjustment in price.	Yes
Spin-off	The spin-off is added to the index on the ex-date at a price of zero. Each individual index methodology will specify whether the spin-off will be eligible to remain in the index beyond the ex-date.	No
Change in IWF	No IWFs are used in price weighted indices.	N/A
Special dividend	When a company pays a special dividend the share price is assumed to drop by the amount of the dividend; the divisor adjustment reflects this drop in index market value.	Yes
Rights offering	Each shareholder receives the right to buy a proportional number of additional shares at a set (often discounted) price. The calculation assumes that the offering is fully subscribed. Divisor adjustment reflects change in price as a result of the rights offering.	Yes

# Equal Weighted Indices

## Definition

An equal weighted index is one where every stock, or company, has the same weight in the index, and a portfolio that tracks the index will invest an equal dollar amount in each applicable instrument. As stock prices move, the weights will shift and exact equality will be lost. Therefore, an equal weighted index must be rebalanced from time to time to re-establish the proper weighting.<sup>2</sup>

The overall approach to calculate equal weighted indices is the same as in the cap-weighted indices; however, the constituents' market values are re-defined to be values that will achieve equal weighting at each rebalancing. Recall two basic formulae:

$$\text{Index Level} = \frac{\text{Index Market Value}}{\text{Divisor}} \quad (13)$$

and

$$\text{Index Market Value} = \sum_i P_i * \text{Shares}_i * \text{IWF}_i \quad (14)$$

To calculate an equal weighted index, the market capitalization for each stock used in the calculation of the index is redefined so that each index constituent has an equal weight in the index at each rebalancing date. In addition to being the product of the stock price, the stock's shares outstanding, and the stock's float factor (IWF), as written above – and the exchange rate when applicable – a new adjustment factor is also introduced in the market capitalization calculation to establish equal weighting.

$$\text{Adjusted Stock Market Value}_i = P_i * \text{Shares}_i * \text{IWF}_i * \text{FxRate}_i * \text{AWF}_i \quad (15)$$

where  $\text{AWF}_i$  (Additional Weight Factor) is the adjustment factor of stock  $i$  assigned at each index rebalancing date,  $t$ , which makes all index constituents modified market capitalization equal (and, therefore, equal weight), while maintaining the total market value of the overall index. The  $\text{AWF}$  for each index constituent,  $i$ , at rebalancing date,  $t$ , is calculated by:

$$\text{AWF}_{i,t} = \frac{Z}{N * \text{FloatAdjustedMarketValue}_{i,t}} \quad (16)$$

where  $N$  is the number of stocks in the index and  $Z$  is an index specific constant set for the purpose of deriving the  $\text{AWF}$  and, therefore, each stock's share count used in the index calculation (often referred to as modified index shares).

The index divisor is defined based on the index level and market value from equation (13). The index level is not altered by index rebalancings. However, since prices and outstanding shares will have changed since the last rebalancing, the divisor will change at the rebalancing.

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<sup>2</sup> In contrast, a cap-weighted index requires no rebalancing as long as there aren't any changes to share counts, IWFs, returns of capital, or stocks added or deleted.

So:

$$(\text{Divisor})_{\text{after rebalancing}} = \frac{(\text{Index Market Value})_{\text{after rebalancing}}}{(\text{Index Value})_{\text{before rebalancing}}} \quad (16a)$$

where:

$$\text{Index Market Value} = \sum_i P_i * \text{Shares}_i * \text{IWF}_i * \text{FxRate}_i * \text{AWF}_i \quad (16b)$$

## Modified Equal Weighted Indices

There are some equal weighted indices that place further restrictions on stocks included in the index. An example restriction might be a cap on the weight allocated to one sector or a cap on the weight of a single country or region in the index. The rules could also stipulate a maximum weight for a stock if the index applies additional liquidity factors (e.g. basket liquidity) when determining the index weights. In any of these situations, if a cap is applied to satisfy the restrictions, the excess weight leftover by the cap would be distributed equally amongst the uncapped companies.

## Corporate Actions and Index Adjustments

The following table shows the necessary adjustments to the index and divisor for managing an equal weighted index. One key issue is how to handle events when one stock is replaced by another. Given that stock prices move all the time, the index is only truly equally weighted at the rebalancing. Therefore, when stocks are added or deleted either the new stock must assume the actual weight of the old stock or the entire index must be rebalanced. However, this is not always the case and may vary by index family.

*For more information on the treatment of corporate actions, please refer to S&P Dow Jones Indices' Equity Indices Policies & Practices document. For more information on the specific treatment within an index family, please refer to that index methodology.*

## Index Actions

Index Action	Adjustment Made to Index	Divisor Adjustment?
Spin-off	The spin-off is added to the index on the ex-date at a price of zero. Each individual index methodology will specify whether the spin-off will be eligible to remain in the index beyond the ex-date. If the spin-off is dropped, the weight of the dropped stock will be reinvested in the parent stock.	No
Rights offering	The price is adjusted to the Price of the Parent Company minus (the Price of Rights Offering/Rights Ratio). The adjustment factor changes according to equation (16), to maintain the weight to be the same as the company had before the rights offering.	No
Stock split	Shares are multiplied by and the price is divided by the split factor.	No
Share/IWF changes	None. The adjustment factor is changed to keep the index weight the same.	No
Special dividends	The price of the stock making the special dividend payment is reduced by the per share special dividend amount after the close of trading on the day before the ex-date.	Yes
Constituent change – even number of adds and drops	The company entering the index goes in at the weight of the company coming out. This weight is used to compute the adjusted weight factor of the added stock, using equation (15). If a company is being removed at a price of 0.00, the replacement goes in at the weight of the deleted company at the close on the day before the effective date. If more than one company is being replaced in the index on a single date, the replacements would be in the order specified.	No, except for the scenario where a company is removed at a price of zero and replacement enters the index at the weight of the deleted company prior to the zero price.
Constituent change – deletion only	The weights of all stocks in the index will change, due to the absolute change in the number of index constituents. Relative weights will stay the same.	Yes

# Modified Market Capitalization Weighted Indices

## Definition

A modified market cap weighted index is one where index constituents have a user-defined weight in the index. Between index rebalancings, corporate actions generally have no effect on index weights, as they are fixed through the processes defined below. As stock prices move, the weights will shift and the modified weights will change. Therefore, as in the case of an equal-weighted index, a modified market cap weighted index must be rebalanced from time to time to re-establish the proper weighting.

The overall approach to calculate modified market cap weighted indices is the same as in the cap-weighted indices; however, the constituents' market values are re-defined to be values that will achieve the user-defined weighting at each rebalancing. Recall two basic formulae:

$$\text{Index Level} = \frac{\text{Index Market Value}}{\text{Divisor}} \quad (17)$$

and

$$\text{Index Market Value} = \sum_i P_i * \text{Shares}_i * \text{IWF}_i * \text{FxRate} \quad (18)$$

To calculate a modified market cap weighted index, the market capitalization for each stock used in the calculation of the index is redefined so that each index constituent has the appropriate user-defined weight in the index at each rebalancing date.

In addition to being the product of the stock price, the stock's shares outstanding, and the stock's float factor (IWF), as written above – and the exchange rate when applicable – a new adjustment factor is also introduced in the market capitalization calculation to establish the appropriate weighting.

$$\text{AdjustedStock Market Value}_i = P_i * \text{Shares}_i * \text{IWF}_i * \text{FxRate}_i * \text{AWF}_i \quad (19)$$

where  $\text{AWF}_i$  is the adjustment factor of stock  $i$  assigned at each index rebalancing date,  $t$ , which adjusts the market capitalization for all index constituents to achieve the user-defined weight, while maintaining the total market value of the overall index.

The  $\text{AWF}$  for each index constituent,  $i$ , on rebalancing date,  $t$ , is calculated by:

$$\text{AWF}_{i,t} = \frac{Z}{\text{FloatAdjustedMarketValue}_{i,t}} * W_{i,t} \quad (20)$$

where  $Z$  is an index specific constant set for the purpose of deriving the  $\text{AWF}$  and, therefore, each stock's share count used in the index calculation (often referred to as modified index shares).  $W_{i,t}$  is the user-defined weight of stock  $i$  on rebalancing date  $t$ .

The index divisor is defined based on the index level and market value from equation (17). The index level is not altered by index rebalancings. However, since prices and outstanding shares will have changed since the last rebalancing, the divisor will change at the rebalancing.

So:

$$(\text{Divisor})_{\text{after rebalancing}} = \frac{(\text{Index Market Value})_{\text{after rebalancing}}}{(\text{Index Value})_{\text{before rebalancing}}} \quad (20a)$$

where:

$$\text{Index Market Value} = \sum_i P_i * \text{Shares}_i * \text{IWF}_i * \text{FxRate}_i * \text{AWF}_i \quad (20b)$$

## Corporate Actions and Index Adjustments

The following table shows the necessary adjustments to the index and the divisor for managing a modified market cap weighted index.

*For more information on the treatment of corporate actions, please refer to S&P Dow Jones Indices' Equity Indices Policies & Practices document. For more information on the specific treatment within an index family, please refer to that index methodology.*

### Index Actions

Index Action	Adjustment Made to Index	Divisor Adjustment?
Spin-off	The spin-off is added to the index on the ex-date at a price of zero. Each individual index methodology will specify whether the spin-off will be eligible to remain in the index beyond the ex-date.	No
Rights offering	The price is adjusted to the Price of the Parent Company minus (the Price of Rights Offering/Rights Ratio). The adjustment factor changes according to equation (20), to maintain the weight to be the same as the company had before the rights offering.	No
Stock split	Shares are multiplied by and the price is divided by the split factor.	No
Share/IWF changes	None. The adjustment factor is changed to keep the index weight the same.	No
Special dividends	The price of the stock making the special dividend payment is reduced by the per share special dividend amount after the close of trading on the day before the ex-date.	Yes.
Merger or acquisition	If the surviving company is already an index member, it is retained in the index. If the surviving company does not meet index criteria, it is removed.	Yes, if there is a removal.
Constituent change	The company entering the index goes in at the weight of the company coming out.	No
Delisting, acquisition or any other corporate action resulting in a constituent deletion.	The stock is dropped from the Index	Yes

# Capped Market Capitalization Indices

## Definition

A capped market cap weighted index is one where single index constituents or defined groups of index constituents are confined to a maximum weight and the excess weight is distributed proportionately among the remaining index constituents. As stock prices move, the weights will shift and the modified weights will change. Therefore, as in the case of an equal-weighted index and a modified market cap index, a capped market cap weighted index must be rebalanced from time to time to re-establish the proper weighting. The methodology for capped indices proceeds similarly to that for modified market cap weighted indices. The main difference between the two methods is the treatment of corporate actions between rebalancing periods. For modified market cap weighted indices most corporate actions which affect the market capitalization of a given stock are counterbalanced by a corresponding change in the AWF assigned to the stock in that index, thus resulting in no weight change to the stock and no index divisor change (see the prior section for reference and the index methodology for further details). On the other hand, for capped indices no AWF change is made due to corporate actions between rebalancings and, thus, the weights of stocks in the index as well as the index divisor will change due to corporate actions.

The overall approach to calculate capped market cap weighted indices is the same as in the pure market-cap weighted indices; however, the constituents' market values are re-defined to be values that will meet the particular capping rules of the index in question. Recall equations 17, 18 and 19 from the prior section:

$$\text{Index Level} = \frac{\text{Index Market Value}}{\text{Divisor}} \quad (17)$$

and

$$\text{Index Market Value} = \sum_i P_i * \text{Shares}_i * IWF_i * \text{FxRate} \quad (18)$$

To calculate a capped market cap weighted index, the market capitalization for each stock used in the calculation of the index is redefined so that each index constituent has the appropriate weight in the index at each rebalancing date.

In addition to being the product of the stock price, the stock's shares outstanding, and the stock's float factor (IWF), as written above – and the exchange rate when applicable – a new adjustment factor is also introduced in the market capitalization calculation to establish the appropriate weighting.

$$\text{Adjusted Stock Market Value}_i = P_i * \text{Shares}_i * IWF_i * \text{FxRate}_i * AWF_i \quad (19)$$

where  $AWF_i$  is the adjustment factor of stock  $i$  assigned at each index rebalancing date,  $t$ , which adjusts the market capitalization for all index constituents to achieve the user-defined weight, while maintaining the total market value of the overall index.

The  $AWF$  for each index constituent,  $i$ , on rebalancing date,  $t$ , is calculated by:

$$AWF_{i,t} = \frac{CW_{i,t}}{W_{i,t}} \quad (21)$$

where  $W_{i,t}$  is the uncapped weight of stock  $i$  on rebalancing date  $t$  based on the float market capitalization of all index constituents; and  $CW_{i,t}$  is the capped weight of stock  $i$  on rebalancing date  $t$  as determined by the capping rule of the index in question and the process for determining capped weights as described in Different Capping Methods below.

The index divisor is defined based on the index level and market value from equation (17). The index level is not altered by index rebalancings. However, since prices and outstanding shares will have changed since the last rebalancing, the divisor will change at the rebalancing.

So:

$$(\text{Divisor})_{\text{after rebalancing}} = \frac{(\text{Index Market Value})_{\text{after rebalancing}}}{(\text{Index Value})_{\text{before rebalancing}}} \quad (21a)$$

where:

$$\text{Index Market Value} = \sum_i P_i * \text{Shares}_i * IWF_i * FxRate_i * AWF_i \quad (21b)$$

## Corporate Actions and Index Adjustments

All corporate actions for capped indices affect the index in the same manner as in market cap weighted indices. For details on how each corporate action is treated please refer to the table for Capitalization Weighted Indices section above.

*For more information on the treatment of corporate actions, please refer to S&P Dow Jones Indices' Equity Indices Policies & Practices document.*

## Different Capping Methods

Capped indices arise due to the need for benchmarks which comply with diversification rules, and in the case of funds and listed products, when the general desire is for a benchmark which is highly concentrated in one or a small number of stocks. Capping may apply to single stock concentration limits or concentration limits on a defined group of stocks. At times, companies may also be represented in an index by multiple share class lines. In these instances, maximum weight capping will be based on company float-adjusted market capitalization, with the weight of multiple class companies allocated proportionally to each share class line based on its float-adjusted market capitalization as of the rebalancing reference date. The standard S&P Dow Jones Indices methodologies for determining the weights of capped indices using the most popular capping methods are described below.

**Single Company Capping.** In a single company capping methodology, no company in an index is allowed to breach a certain pre-determined weight as of each rebalancing period. The procedure for assigning capped weights to each company at each rebalancing is as follows:

1. With data reflected on the rebalancing reference date, each company is weighted by float-adjusted market capitalization.
2. If any company has a weight greater than X% (where X% is the maximum weight allowed in the index), that company has its weight capped at X%.
3. All excess weight is proportionally redistributed to all uncapped companies within the index.
4. After this redistribution, if the weight of any other company(s) then breaches X%, the process is repeated iteratively until no companies breach the X% weight cap.

**Single Company and Concentration Limit Capping.** In a single company and concentration limit capping methodology, no company in an index is allowed to breach a certain pre-determined weight and all companies with a weight greater than a certain amount are not allowed, as a group, to exceed a pre-determined total weight. One example of this is 4.5%/22.5%/45% capping (B/A/C in the following

example). No single company is allowed to exceed 22.5% of the index and all companies with a weight greater than 4.5% of the index cannot exceed, as a group, 45% of the index. The procedure for assigning capped weights to each company at each rebalancing is as follows:

1. With data reflected on the rebalancing reference date, each company is weighted by float-adjusted market capitalization.
2. If any company has a weight greater than A% (where A% is the maximum weight allowed in the index), that company has its weight capped at A%.
3. All excess weight is proportionally redistributed to all uncapped companies within the index.
4. After this redistribution, if the weight of any other company(s) then breaches A%, the process is repeated iteratively until no companies breach the A% weight cap.
5. The sum of the companies with weight greater than B% cannot exceed C% of the total weight.
6. If the rule in step 5 is breached, all the companies are ranked in descending order of their weights and the company with the lowest weight that causes the C% limit to be breached is identified. The weight of this company is, then, reduced either until the rule in step 5 is satisfied or it reaches B%.
7. This excess weight is proportionally redistributed to all companies with weights below B%. Any stock that receives weight cannot breach the B% cap. This process is repeated iteratively until step 5 is satisfied or until all stocks are greater than or equal to B%.
8. If the rule in step 5 is still breached and all stocks are greater than or equal to B%, the company with the lowest weight that causes the C% limit to be breached is identified. The weight of this company is, then, reduced either until the rule in step 5 is satisfied or it reaches B%.
9. This excess weight is proportionally redistributed to all companies with weights greater than B%. Any stock that receives weight cannot breach the A% stock cap. This process is repeated iteratively until step 5 is satisfied.

For indices that use capping rules across more than one attribute, S&P Dow Jones Indices will utilize an optimization program to satisfy the capping rules. The stated objective for the optimization will be to minimize the difference between the pre-capped weights of the stocks in the index and the final capped weights.

# Pure Style Indices

For the S&P Pure Style Indices, introduced in 2005, a stock's weight depends on its growth or value attribute measurements, the same measures that are used in the index stock selection process. The discussion here covers how these indices are calculated; the selection of stocks is covered in the *S&P U.S. Style Indices methodology*.

There are both Pure Growth Style and Pure Value Style indices. Under the selection process, each stock has a growth score and a value score. These scores are used to identify pure growth stocks and pure value stocks.<sup>3</sup> The Pure Growth index includes only pure growth stocks; a stock's weight in the index is determined by its growth score; likewise for pure value.

For more information on the calculation of S&P Pure Style indices, please refer to the *S&P U.S. Style Indices Methodology* located on our website, [www.spdji.com](http://www.spdji.com).

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<sup>3</sup> A stock cannot be both pure growth and pure value; it can be neither pure growth nor pure value.

# Weighted Return Indices

S&P Dow Jones Indices' Weighted Return Indices combine the returns of two or more underlying indices using a specified set of weighting rules to create a new unique index return series. An index that uses the Weighted Return methodology might also be referred to as an "Index of Indices."

On any trading date,  $t$ , the index is calculated as follows using the component indices as detailed on the prior pages:

$$Index_t = Index_{PB} * (1 + IndexReturn_t) \quad (22)$$

$$IndexReturn_t = \sum_{i=1}^n w_i R_i \quad (23)$$

where:

$Index_{PB}$  = Index value on the previous rebalancing date

$w_i$  = Weight of an asset class  $i$

$R_i$  = Cumulative return of the representative asset class  $i$  at  $t$  from the previous rebalancing date

# Leveraged and Inverse Indices

## Leveraged Indices for Equities

S&P Dow Jones Indices' Leveraged Indices are designed to generate a multiple of the return of the underlying index in situations where the investor borrows funds to generate index exposure beyond his/her cash position. The approach is to first calculate the underlying index, then calculate the daily returns for the leveraged index and, finally, to calculate the current value of the leveraged index by incrementing the previous value by the daily return. There is no change to the calculation of the underlying index.

The daily return for the leveraged index consists of two components: (1) the return on the total position in the underlying index less (2) the borrowing costs for the leverage.

The formula for calculating the Leveraged Index is as follows:

$$\text{Leveraged Index Return} = K * \left( \frac{\text{Underlying Index}_t}{\text{Underlying Index}_{t-1}} - 1 \right) - (K - 1) * \left( \frac{\text{Borrowing Rate}}{360} \right) * D_{t,t-1} \quad (24)$$

In equation (24) the borrowing rate is applied to the leveraged index value because this represents the funds being borrowed. Given this, the Leveraged Index Value at time  $t$  can be calculated as:

$$\text{Leveraged Index Value}_t = (\text{Leveraged Index Value}_{t-1}) * (1 + \text{Leveraged Index Return}) \quad (25)$$

Substituting (24) into (25) and expanding the right hand side of (25) yields:

$$\begin{aligned} \text{Leveraged Index Value}_t = \\ \text{Leveraged Index Value}_{t-1} * \left[ 1 + \left[ K * \left( \frac{\text{Underlying Index}_t}{\text{Underlying Index}_{t-1}} - 1 \right) - (K - 1) * \left[ \frac{\text{Borrowing Rate}}{360} \right] * D_{t,t-1} \right] \right] \end{aligned} \quad (26)$$

where:

$K (K \geq 1)$  = Leverage Ratio

- $K = 1$ , no leverage
- $K = 2$ , Exposure = 200%
- $K = 3$ , Exposure = 300%

*Borrowing Rate* = Overnight LIBOR in the U.S. or EONIA in Europe are two common examples

$D_{t,t-1}$  = the number of calendar days between date  $t$  and  $t-1$

In the absence of leverage ( $K=1$ ),

$$\text{Leveraged Index Value}_t = \text{Leveraged Index Value}_{t-1} * \left[ \frac{\text{Underlying Index}_t}{\text{Underlying Index}_{t-1}} \right] \quad (27)$$

The leverage position is rebalanced daily. This is consistent with the payoff from futures based replication.

## Leveraged Indices without Borrowing Costs for Equities

In some cases, leveraged indices that do not account for costs incurred to finance the associated leverage are calculated. For these indices, the borrowing rate in formulas (24) and (26) is set to zero and the calculation follows as above.

## Inverse Indices for Equities

S&P Dow Jones Indices' Inverse indices are designed to provide the inverse performance of the underlying index; this represents a short position in the underlying index. The calculation follows the same general approach as the leveraged index with certain adjustments: First, the return on the underlying index is reversed. Second, while the costs of borrowing the securities are not included, there is an adjustment to reflect the interest earned on both the initial investment and the proceeds from selling short the securities in the underlying index. These assumptions reflect normal industry practice.<sup>4</sup>

The general formula for the return to the inverse index is

$$\begin{aligned} \text{Inverse Index Return} = & -K * \left( \frac{\text{Underlying Index}_t}{\text{Underlying Index}_{t-1}} - 1 \right) \\ & + (K + 1) * \left( \frac{\text{Lending Rate}}{360} \right) D_{t,t-1} \end{aligned} \quad (28)$$

Where the first right hand side term represents the return on the underlying index and the second right hand side term represents the interest earned on the initial investment and the shorting proceeds.

Expanding this as done above for the leveraged index yields:

$$\begin{aligned} \text{Inverse Index Value}_t = \\ \text{Inverse Index Value}_{t-1} * \left[ 1 - \left[ K * \left( \frac{\text{Underlying Index}_t}{\text{Underlying Index}_{t-1}} - 1 \right) - (K + 1) * \left[ \frac{\text{Lending Rate}}{360} \right] * D_{t,t-1} \right] \right] \end{aligned} \quad (29)$$

where:

$K (K \geq 1)$  = Leverage Ratio

- $K = 1$ , Exposure = -100%
- $K = 2$ , Exposure = -200%
- $K = 3$ , Exposure = -300%

*Lending Rate* = Overnight LIBOR in the U.S. or EONIA in Europe are two common examples

$D_{t,t-1}$  = the number of calendar days between date  $t$  and  $t-1$

In the absence of leverage ( $K = 1$ ),

$$\begin{aligned} \text{Inverse Index Value}_t = \\ \text{Inverse Index Value}_{t-1} * \left[ 1 - \left[ \left( \frac{\text{Underlying Index}_t}{\text{Underlying Index}_{t-1}} - 1 \right) - (2) * \left[ \frac{\text{Lending Rate}}{360} \right] * D_{t,t-1} \right] \right] \end{aligned} \quad (30)$$

The inverse position is rebalanced daily. This is consistent with the payoff from futures based replication.

<sup>4</sup> Straightforward adjustments can be made to either to include the costs of borrowing securities or to exclude the interest earned on the shorting proceeds and the initial investment.

## Inverse Indices without Borrowing Costs for Equities

In some cases, inverse indices that do not account for any interest earned are calculated. For these indices, the lending rate in formulas (28) and (29) is set to zero and the calculation follows as above.

## Leveraged and Inverse Indices for Futures

S&P Dow Jones Indices' futures-based Leveraged Indices are designed to generate a multiple of the return of the underlying futures index in situations where the investor borrows funds to generate index exposure beyond his/her cash position.

S&P Dow Jones Indices' futures-based Inverse indices are designed to provide the inverse performance of the underlying futures index; this represents a short position in the underlying index.

The approach is to first calculate the underlying index, then calculate the daily returns for the leveraged or inverse index. There is no change to the calculation of the underlying futures index.

The leveraged or inverse index may be rebalanced daily or periodically.

## Daily Rebalanced Leverage or Inverse Futures Indices

If the S&P Dow Jones Indices futures-based leveraged or inverse index is rebalanced daily, the index excess return is the multiple of the underlying index's excess return and calculated as follows:

$$IndexER_t = IndexER_{t-1} * \left( 1 + \left( K * \left( \frac{UnderlyingIndexER_t}{UnderlyingIndexER_{t-1}} - 1 \right) \right) \right) \quad (31)$$

where:

$K$  ( $K \neq 0$ ) = Leverage/Inverse Ratio

- $K = 1$ , no leverage
- $K = 2$ , leverage exposure = 200%
- $K = 3$ , leverage exposure = 300%
- $K = -1$ , inverse exposure = -100%

A total return version of each of the indices is calculated, which includes interest accrual on the notional value of the index based on a specified interest rate (e.g. 91-day U.S. Treasury rate), as follows:

$$IndexTR_t = IndexTR_{t-1} * \left( \left( \frac{IndexER_t}{IndexER_{t-1}} \right) + TBR_t \right) \quad (32)$$

where:

$IndexTR_{t-1}$  = The Index Total Return on the preceding business day

$TBR_t$  = Treasury Bill Return, as determined by the following formula:

$$TBR_t = \left[ \frac{1}{1 - \frac{91}{360} * TBAR_{t-1}} \right]^{\frac{Delta_t}{91}} - 1 \quad (33)$$

$Delta_t$  = The number of calendar days between the current and previous business days

$TBAR_{t-1}$  = The most recent weekly high discount rate for 91-day U.S. Treasury bills effective on the preceding business day<sup>5</sup>

### Periodically Rebalanced Leverage or Inverse Futures Indices

If the S&P Dow Jones Indices futures-based leveraged or inverse index is rebalanced periodically (e.g. weekly, monthly, or quarterly), the index excess return is the multiple of the underlying index excess return since last rebalancing business day and shall be calculated as follows:

$$IndexER_t = IndexER_{t\_LR} * \left( 1 + \left( K * \left( \frac{UnderlyingIndexER_t}{UnderlyingIndexER_{t\_LR}} - 1 \right) \right) \right) \quad (34)$$

where:

$IndexER_{t\_LR}$  = The Index Excess Return on the last rebalancing business day,  $t\_LR$

$UnderlyingIndexER_{t\_LR}$  = The Underlying Index Excess Return value on the last rebalancing business day,  $t\_LR$

$t\_LR$  = The last rebalancing business day

$K (K \neq 0)$  = Leverage / Inverse Ratio

- $K = 1$ , no leverage
- $K = 2$ , leverage exposure = 200%
- $K = 3$ , leverage exposure = 300%
- $K = -1$ , inverse exposure = -100%

A total return version of each of the indices is calculated, which includes interest accrual on the notional value of the index based on the 91-day U.S. Treasury rate. The formulae are the same as (32) and (33) above.

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<sup>5</sup> Generally the rates are announced by the U.S. Treasury on each Monday. On Mondays that are bank holidays, Friday's rates apply.

# Total Return Calculations

The preceding discussions were related to price indices where changes in the index level reflect changes in stock prices. In a total return index changes in the index level reflect both movements in stock prices and the reinvestment of dividend income. A total return index represents the total return earned in a portfolio that tracks the underlying price index and reinvests dividend income in the overall index, not in the specific stock paying the dividend.

The total return construction differs from the price index and builds the index from the price index and daily total dividend returns. The first step is to calculate the total dividend paid on a given day and convert this figure into points of the price index:

$$TotalDailyDividend = \sum_i Dividend_i * Shares_i \quad (35)$$

Where *Dividend* is the dividend per share paid for stock *i* and *Shares* are the shares. This is done for each trading day. *Dividend<sub>i</sub>* is generally zero except for four times a year when it goes ex-dividend for the quarterly dividend payment. Stocks may also issue dividends on a monthly, semi-annual or annual basis. Some stocks do not pay a dividend and *Dividend* is always zero. *TotalDailyDividend* is measured in dollars. This is converted to index points by dividing by the divisor for the underlying price index:

$$IndexDividend = \frac{TotalDailyDividend}{Divisor} \quad (36)$$

The next step is to apply the usual definition of a total return from a financial instrument to the price index. Equation (35) gives the definition, and equation (36) applies it to the index:

$$TotalReturn = \left( \frac{P_t + D_t}{P_{t-1}} \right) - 1 \quad (37)$$

and

$$DTR_t = \left( \frac{IndexLevel_t + IndexDividend_t}{IndexLevel_{t-1}} - 1 \right) \quad (38)$$

where the *TotalReturn* and the daily total return for the index (*DTR*) is stated as a decimal. The *DTR* is used to update the total return index from one day to the next:

$$TotalReturnIndex_t = (TotalReturnIndex_{t-1}) * (1 + DTR_t) \quad (39)$$

# Net Total Return Calculations

To account for tax withheld from dividends, a net total return calculation is used. The calculation is identical to the calculations detailed in the previous Total Return section, except each dividend is adjusted to account for the tax taken out of the payment.

Inserting the withholding rate into the calculation at the first step is all that needs to be done – the calculation can follow identically from that point forward:

$$TotalDailyDividend = \sum_i Dividend_i * Shares_i * (1 - WithholdingRate_i) \quad (40)$$

The tax rates used for S&P Dow Jones Indices' global indices are from the perspective of a Luxembourg investor. However, in domestic index families, tax rates from the perspective of a domestic investor will be applied.

# Franking Credit Adjusted Total Return Indices

Additional total return indices are available for a number of S&P/ASX Indices that adjust for the tax effect of franking credits attached to cash dividends. The indices utilize tax rates relevant to two segments of investors: one version incorporates a 0% tax rate relevant for tax-exempt investors and a second version uses a 15% tax rate relevant for superannuation funds. The franking credits attached to both regular and special cash dividends are included in the respective calculations.

To calculate the gross dividend points reinvested in the Franking Credit Adjusted Total Return Indices:

$$\text{Grossed-up Dividend} = [\text{As Reported Dividend} * (1 - \% \text{ Franked}) + (\text{As Reported Dividend} * \% \text{ Franked} / (1 - \text{Company Tax Rate}))] \quad (41)$$

The Net Tax Effect of the franking credit is then calculated based on the investor tax rate (i.e. 0% for tax-exempt investors and 15% for superannuation funds).

$$\text{Net Tax Effect} = [\text{Grossed-up Dividend} * (1 - \text{Investor Tax Rate})] - \text{As Reported Dividend} \quad (42)$$

The Net Tax Effect of each dividend is then multiplied by the index shares of that company to calculate the gross dividend market capitalization.

$$\text{Gross Dividend Market Cap} = \text{Net Tax Effect} * \text{Index Shares} \quad (43)$$

These are then summed for all dividends going ex on that date and converted to dividend points by dividing by the index divisor

$$\text{Gross Dividend Points} = \text{Sum of Gross Dividend Market Caps} / \text{Index Divisor} \quad (44)$$

**Franking Credit Adjusted Annual Total Return Indices.** This index series accrues a pool of gross dividend points on a daily basis and reinvests them across the index annually after the end of the financial year. Reinvestment occurs at market close on the first trading day after June 30<sup>th</sup>. The gross dividend points are derived by taking the value of the gross dividend market capitalization (less the as reported dividend market capitalization) and dividing it by the index divisor effective on the ex-date of the respective dividend.

**Franking Credit Adjusted Daily Total Return Indices.** Rather than allowing a separate accrual of gross dividend points, this index series reinvests the gross dividend amount across the index at the close of the ex-date on a daily basis.

# Index Fundamentals

Indices are often used to measure market conditions or gauge valuations among markets or between stocks and indices through measures like earnings per share (EPS), price-earnings ratios, dividend yields and so forth. These are calculated by using the divisor as if it represents shares for a company. The basic format is illustrated for the EPS for an index:

$$Index\ EPS = \frac{\sum_i eps_i * shares_i}{Divisor} \quad (45)$$

where *IndexEPS* is the EPS for the overall index, *eps<sub>i</sub>* is the EPS for stock *i* and *shares<sub>i</sub>* are the shares used to calculate the index with any adjustments such as the IWF incorporated into the figure. If the calculation refers to an equal weighted or attribute weighted index, the calculation use the shares defined for those indices (*C\_shares* or *M\_shares*, as appropriate).

The price-earnings (PE) ratio for the index is simply the ratio of the index level (or price) to the index EPS. For a cap-weighted index, this can also be calculated directly from the stock level data by dividing the total market cap of the index by total earnings of all companies in the index. In this calculation, the Divisor terms in the denominator drop out:

$$IndexPE = \frac{\frac{\sum P_i * Shares_i}{Divisor}}{\frac{\sum eps_i * Shares_i}{Divisor}} \quad (46)$$

The same general approach can be used for various index fundamentals and ratios such as book value per share, price-to-book, dividend-to-price (i.e. dividend yield) and so forth.

# Dividend Indices

S&P Dow Jones Indices' Dividend Indices are designed to track the total dividend payments from the constituents of an underlying index. The level of the index is based on a running total of dividends of the constituents of the underlying index. Some indices reset to zero on a periodic basis, generally quarterly or annually. Thus, the index measures the total dividends paid in the underlying index since the previous rebalancing date, or the base date for indices that do not reset on a periodic basis. For quarterly indices, the index resets to zero after the close on the third Friday of the last month of the quarter, to coincide with futures and options expiration. For annual indices, the index resets to zero after the close on the third Friday of December, to coincide with futures and options expiration.

The formula for calculating the dividend index on any date,  $t$ , for a given underlying index,  $x$ , is:

$$DividendIndex_{t,x} = \sum_{i=r+1}^t ID_{i,x} \quad (47)$$

where:

$ID_{i,x}$  = The index dividend of the underlying index  $x$  on day  $i$ .

$t$  = The current date.

$r+1$  = The trading date immediately following the reset date of the index (or base date if the index does not reset periodically).

The index dividend ( $ID$ ) of the underlying index is calculated on any given day as the total dividend value for all constituents of the index divided by the index divisor. The total dividend value is calculated as the sum of dividends per share multiplied by index shares outstanding for all constituents of the index which have a dividend going ex on the date in question. For more detail concerning the calculation of index dividends please refer to the *Total Returns Calculation* section above.

# Risk Control Indices

S&P Dow Jones Indices' Risk Control Indices are designed to track the return of a strategy that applies dynamic exposure to an underlying index in an attempt to control the level of volatility.

The index includes a leverage factor that changes based on realized historical volatility. If realized volatility exceeds the target level of volatility, the leverage factor will be less than one; if realized volatility is lower than the target level, the leverage factor may be greater than one, assuming the index allows for a leverage factor of greater than one. A given Risk Control Index may have a maximum leverage factor that cannot be exceeded. There are no guarantees that the index shall achieve its stated targets.

The return of the index consists of two components: (1) the return on the position in the underlying index and (2) the interest cost or gain, depending upon whether the position is leveraged or deleveraged.

A leverage factor greater than one represents a leveraged position, a leverage factor equal to one represents an unleveraged position, and a leverage factor less than one represents a deleveraged position. The leverage factor may change periodically, on a set schedule, or may change when volatility exceeds or falls below predetermined volatility thresholds.

For equity indices, the leverage factor will not change at the close of any index calculation day in which stocks representing 15% or more of the total weight of the underlying index are not trading due to an exchange holiday. At each underlying index's rebalancing, and using each stock's weight at that time, a forward looking calendar of such dates is determined and posted on S&P Dow Jones Indices' Web site at [www.spdji.com](http://www.spdji.com).

The formula for calculating the Risk Control Index is as follows:

$$Risk\ Control\ Index\ Return_t = K_{rb} * \left( \frac{Underlying\ Index_t}{Underlying\ Index_{rb}} - 1 \right) + (1 - K_{rb}) * \left[ \prod_{i=rb+1}^t (1 + InterestRate_{i-1} * D_{i-1,i} / 360) - 1 \right] \quad (48)$$

The Risk Control Index Value at time  $t$  can, then, be calculated as:

$$RiskControlIndexValue_t = (RiskControlIndexValue_{rb}) * (1 + RiskControlIndexReturn_t) \quad (49)$$

Substituting equation (48) into (49) and expanding yields:

$$Risk\ Control\ Index\ Value_t = Risk\ Control\ Index\ Value_{rb} * \left[ 1 + \left[ K_{rb} * \left( \frac{Underlying\ Index_t}{Underlying\ Index_{rb}} - 1 \right) + (1 - K_{rb}) * \left[ \prod_{i=rb+1}^t (1 + InterestRate_{i-1} * D_{i-1,i} / 360) - 1 \right] \right] \right] \quad (50)$$

where:

- $UnderlyingIndex_t$  = The level of the underlying index on day  $t$
- $UnderlyingIndex_{rb}$  = The level of the underlying index as of the previous rebalancing date
- $rb$  = The last index rebalancing date<sup>6</sup>
- $K_{rb}$  = The leverage factor set at the last rebalancing date, calculated as:  

$$Min(Max K, Target Volatility/Realized Volatility_{rb-d})$$
- $Max K$  = The maximum leverage factor allowed in the index
- $d$  = The number of days between when volatility is observed and the rebalancing date (e.g. if  $d = 2$ , the historical volatility of the underlying index as of the close two days prior to the rebalancing date will be used to calculate the leverage factor  $K_{rb}$ )
- $Target Volatility$  = The target level of volatility set for the index
- $Realized Volatility_{rb-d}$  = The historical realized volatility of the underlying index as of the close of  $d$  trading days prior to the previous rebalancing date,  $rb$ , where a trading day is defined as a day on which the underlying index is calculated
- $Interest Rate_{i-1}$  = The interest rate set for the index<sup>7</sup>

For indices that replicate a rolling investment in a three-month interest rate the above formula is altered to:

$$Risk\ Control\ Index\ Value_t = \left[ 1 + \left[ K_{rb} * \left( \frac{Underlying\ Index_t}{Underlying\ Index_{rb}} - 1 \right) + (1 - K_{rb}) * \left[ \prod_{i=rb+1}^t (1 + InterestRate_{i-1}) - 1 \right] \right] \right] \quad (51)$$

where:

$$InterestRate_{i-1} = (D_{i-1,i} * IR3M_{i-1} - (IR3M_{i-1} - IR3M_{i-2} - D_{i-1,i} * (IR3M_{i-1} - IR2M_{i-1}) * (1/30)) * 90) / 360$$

where:

- $D_{i-1,t}$  = The number of calendar days between day  $i-1$  and day  $t$
- $IR3M_{i-1}$  = Three-month interest rate on day  $i-1$
- $IR2M_{i-1}$  = Two-month interest rate on day  $i-1$

For indices that are rebalanced daily, the leverage factor is not recalculated at the close of any index calculation day when stocks representing 15% or more of the total weight of the underlying index are not trading due to an exchange holiday. If  $rb$  is a holiday, then  $K_{rb}$  is calculated as follows:

$$K_{rb} = K_{rb-1} * \left( \frac{Underlying\ Index_{rb}}{Underlying\ Index_{rb-1}} \right) / \left( \frac{Risk\ Control\ Index\ Value_{rb}}{Risk\ Control\ Index\ Value_{rb-1}} \right)$$

This shows what the effect will be on  $rb$ , given that no adjustment of positions is allowed to occur on such days. The leverage factor will adjust solely to account for market movements on that day.

For periodically rebalanced risk control indices,  $K_{rb}$  is calculated at each rebalancing and held constant until the next rebalancing.

<sup>6</sup> The inception date of each risk control index is considered the first rebalancing date of that index.

<sup>7</sup> The interest rate may be an overnight rate, such as LIBOR or EONIA, or a daily valuation of a rolling investment in a three-month interest rate, or zero. A 360-day year is assumed for the interest calculations in accordance with U.S. banking practices.

For large position moves, some investors like to rebalance risk control indices intra-period, when the periodicity is longer than daily. This feature is incorporated in the risk-control framework by introducing a barrier,  $K_b$ , on the leverage factor. Intra-period rebalancing is allowed only if the absolute change of the equity leverage factor  $K_t$ , at time  $t$ , is larger than the barrier  $K_b$  from the value at the last rebalancing date.

The equity leverage factor  $K_t$  is calculated as:

$$K_t = \text{Min}(\text{Max } K, \text{Target Volatility/Realized Volatility}_{t-d})$$

If no barrier is provided for the index, then intra-period rebalancing is not allowed.

### Excess Return Indices

S&P Dow Jones Indices' Excess Return Indices are designed to track an unfunded investment in an underlying index. In other words, an excess return index calculates the return on an investment in an index where the investment was made through the use of borrowed funds. Thus the return of an excess return index will be equal to that of the underlying index less the associated borrowing costs. Most S&P Dow Jones Indices calculate an excess return index level to mirror an unfunded position.

The formula for calculating the Excess Return Index is as follows:

$$\text{ExcessReturn} = \left( \frac{\text{Underlying Index}_t}{\text{Underlying Index}_{t-1}} - 1 \right) - \left( \frac{\text{Borrowing Rate}}{360} \right) * D_{t,t-1} \quad (52)$$

The Excess Return Index Value at time  $t$  can be calculated as:

$$\text{ExcessReturn IndexValue}_t = (\text{ExcessReturn Index Value}_{t-1}) * (1 + \text{Excess Return}) \quad (53)$$

Substituting (52) into (53) and expanding the right hand side of (53) yields:

$$\begin{aligned} \text{ExcessReturn IndexValue}_t = \\ \text{ExcessReturn IndexValue}_{t-1} * \left[ 1 + \left[ \left( \frac{\text{Underlying Index}_t}{\text{Underlying Index}_{t-1}} - 1 \right) - \left[ \frac{\text{Borrowing Rate}}{360} \right] * D_{t,t-1} \right] \right] \end{aligned} \quad (54)$$

where:

**Borrowing Rate** = The investment funds borrowing rates, which will differ for each excess return index<sup>8</sup>

**$D_{t,t-1}$**  = The number of calendar days between date  $t$  and  $t-1$

<sup>8</sup> Generally an overnight rate, such as overnight LIBOR in the U.S. or EONIA in Europe, will be used. However, in some cases other interest rates may be used. A 360-day year is assumed for the interest calculations in accordance with U.S. banking practices.

## Exponentially-Weighted Volatility

The realized volatility is calculated as the maximum of two exponentially weighted moving averages, one measuring short-term and one measuring long-term volatility.

$$RealizedVolatility_t = \text{Max}(RealizedVolatility_{S,t}, RealizedVolatility_{L,t})$$

where:

$S,t$  = The short-term volatility measure at time  $t$ , calculated as:

(55)

$$RealizedVolatility_{S,t} = \sqrt{\frac{252}{n} * Variance_{S,t}}$$

for  $t > T_0$

$$Variance_{S,t} = \lambda_S * Variance_{S,t-1} + (1 - \lambda_S) * \left[ \ln\left(\frac{UnderlyingIndex_t}{UnderlyingIndex_{t-n}}\right) \right]^2$$

for  $t = T_0$

$$Variance_{S,T_0} = \sum_{i=m+1}^{T_0} \frac{\alpha_{S,i,m}}{WeightingFactor_S} * \left[ \ln\left(\frac{UnderlyingIndex_i}{UnderlyingIndex_{i-n}}\right) \right]^2$$

$L,t$  = The long-term volatility measure at time  $t$ , calculated as:

(56)

$$RealizedVolatility_{L,t} = \sqrt{\frac{252}{n} * Variance_{L,t}}$$

for  $t > T_0$

$$Variance_{L,t} = \lambda_L * Variance_{L,t-1} + (1 - \lambda_L) * \left[ \ln\left(\frac{UnderlyingIndex_t}{UnderlyingIndex_{t-n}}\right) \right]^2$$

for  $t = T_0$

$$Variance_{L,T_0} = \sum_{i=m+1}^{T_0} \frac{\alpha_{L,i,m}}{WeightingFactor_L} * \left[ \ln\left(\frac{UnderlyingIndex_i}{UnderlyingIndex_{i-n}}\right) \right]^2$$

where:

- $T_0$  = The start date for a given risk control index
- $n$  = the number of days inherent in the return calculation used for determining volatility<sup>9</sup>
- $m$  = the  $N^{\text{th}}$  trading date prior to  $T_0$
- $N$  = the number of trading days observed for calculating the initial variance as of the start date of the index
- $\lambda_S$  = The short-term decay factor used for exponential weighting<sup>10</sup>
- $\lambda_L$  = The long-term decay factor used for exponential weighting<sup>10</sup>

<sup>9</sup> If  $n = 1$  daily returns are used, while if  $n = 2$  two day returns are used, and so forth.

<sup>10</sup> The decay factor is a number greater than zero and less than one that determines the weight of each daily return in the calculation of historical variance.

$\alpha_{S,m,i}$  = Weight of date  $t$  in the short-term volatility calculation, as calculated based on the following formula:

$$\alpha_{S,t} = (1 - \lambda_S) * \lambda_S^{N+m-i}$$

$$WeightingFactor_S = \sum_{i=m+1}^{T_0} \alpha_{S,i,m}$$

$\alpha_{L,m,i}$  = Weight of date  $t$  in the long-term volatility calculation, as calculated based on the following formula:

$$\alpha_{L,t} = (1 - \lambda_L) * \lambda_L^{N+m-i}$$

$$WeightingFactor_L = \sum_{i=m+1}^{T_0} \alpha_{L,i,m}$$

The interest rate, maximum leverage, target volatility and the lambda decay factors are defined in relation to each index and are generally held constant throughout the life of the index. The leverage position changes at each rebalancing based on changes in realized volatility. There is a two-day lag between the calculation of the leverage factor, based on the ratio of target volatility to realized volatility, and the implementation of that leverage factor in the index.

The above formulae can be used for simpler models by the appropriate choice of parameters. For example, if the short-term and long-term decay factors,  $\lambda_S$  and  $\lambda_L$  are set to the same value (e.g. 5%) then there are no separate considerations for short-term and long-term volatility.

### Simple-Weighted Volatility

The realized volatility is calculated as the maximum of two simple-weighted moving averages, one measuring short-term volatility and one measuring long-term volatility.

$$RealizedVolatility_t = \text{Max}(RealizedVolatility_{S,t}, RealizedVolatility_{L,t})$$

where:

$S,t$  = The short-term volatility measure at time  $t$ , calculated as:

(57)

$$RealizedVolatility_{S,t} = \sqrt{\frac{252}{n} * Variance_{S,t}}$$

$$Variance_{S,t} = 1/N_S * \sum_{i=t-N_S+1}^t \ln\left(\frac{UnderlyingIndex_i}{UnderlyingIndex_{i-n}}\right)^2$$

$L,t$  = The long-term volatility measure at time  $t$ , calculated as: (58)

$$RealizedVolatility_{L,t} = \sqrt{\frac{252}{n} * Variance_{L,t}}$$

$$Variance_{L,t} = 1/N_L * \sum_{i=t-N_L+1}^t \ln\left(\frac{UnderlyingIndex_i}{UnderlyingIndex_{i-n}}\right)^2$$

where:

- $n$  = The number of days inherent in the return calculation used for determining volatility<sup>11</sup>
- $N_S$  = The number of trading days observed for calculating variance for the short-term volatility measure
- $N_L$  = The number of trading days observed for calculating variance for the long-term volatility measure

### Futures-Based Risk Control Indices

When the underlying index is based on futures contracts, most of the Risk Control methodology follows the details on the prior six pages. However, there are some differences as detailed below, particularly as it relates to the cash component of the index.

For such an index, it includes a leverage factor that changes based on realized historical volatility. If realized volatility exceeds the target level of volatility, the leverage factor will be less than one; if realized volatility is lower than the target level, the leverage factor may be greater than one. A given risk control index may have a maximum leverage factor that cannot be exceeded.

For equity risk control indices, the return consists of two components: (1) the return on the position in the underlying S&P Dow Jones Indices index and (2) the interest cost or gain, depending upon whether the position is leveraged or deleveraged. For futures-based risk control indices, there is no borrowing or lending to achieve investment objectives in the underlying index. Therefore, the cash component of the Index does not exist.

Again, a leverage factor greater than one represents a leveraged position, a leverage factor equal to one represents an unleveraged position, and a leverage factor less than one represents a deleveraged position. The leverage factor may change at regular intervals, in response to changes in realized historical volatility, or when the expected volatility exceeds or falls below predetermined volatility thresholds, if such thresholds were in place.

The formula for calculating the Risk Control Excess Return Index largely follows that detailed beginning with equation (48). However, since there is no funding for such indices (as opposed to the case with equity excess return indices, where it is assumed the initial investment is borrowed and excess cash is invested), the interest rate used in the calculation is eliminated:

$$Risk\ Control\ Excess\ Return\ Index\ Return_t = K_{rb} * \left( \frac{Underlying\ Index_t}{Underlying\ Index_{rb}} - 1 \right) \tag{59}$$

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<sup>11</sup> If  $n = 1$  daily returns are used, while if  $n = 2$  two day returns are used, and so forth.

The Risk Control Excess Return Index Value at time  $t$  can, then, be calculated as:

$$\begin{aligned} RiskControlExcessReturnIndexValue_t = \\ (RiskControlExcessReturnIndexValue_{rb}) * (1 + RiskControlExcessReturnIndexReturn_t) \end{aligned} \quad (60)$$

The formula for calculating the Risk Control Total Return Index, which includes interest earned on Treasury Bills, is as follows:

$$\begin{aligned} RiskControlTotalReturnIndexReturn_t = \\ K_{rb} * \left( \frac{UnderlyingIndex_t}{UnderlyingIndex_{rb}} - 1 \right) + \left[ \prod_{i=rb+1}^t (1 + InterestRate_{i-1} * D_{i-1,i} / 360) - 1 \right] \end{aligned} \quad (61)$$

The Risk Control Total Return Index Value at time  $t$  can, then, be calculated as:

$$\begin{aligned} RiskControlTotalReturnIndexValue_t = \\ (RiskControlTotalReturnIndexValue_{rb}) * (1 + RiskControlTotalReturnIndexReturn_t) \end{aligned} \quad (62)$$

Substituting equation (61) into (62) and expanding yields:

$$\begin{aligned} RiskControlTotalReturnIndexValue_t = \\ RiskControlIndexValue_{rb} * \\ \left[ 1 + \left[ K_{rb} * \left( \frac{UnderlyingIndex_t}{UnderlyingIndex_{rb}} - 1 \right) + \left[ \prod_{i=rb+1}^t (1 + InterestRate_{i-1} * D_{i-1,i} / 360) - 1 \right] \right] \right] \end{aligned} \quad (63)$$

where all variables in equations (59)-(63) are the same as those defined for (48)-(50) except:

$$InterestRate_{i-1} = \text{The interest rate set for the index}^{12}$$

### Exponentially-Weighted Volatility for Futures-Based Risk Control Indices

Please refer to pages 34 for information on Exponentially-Weighted Volatility. However, for futures-based risk control indices there is a three (3)-day lag between the calculation of the leverage factor, based on the ratio of target volatility to realized volatility, and the implementation of that leverage factor in the index.

### Dynamic Volatility Risk Control Indices

In dynamic volatility risk control indices, the volatility target is not set as a definition of the index. Rather it is set at various levels based on the moving average of VIX computed over a predetermined number of days (e.g. 30-day moving average).

### Variance Based Risk Control Indices

In variance-based risk control indices, a target level of variance is set rather than a target volatility level. This allows for faster leveraging or deleveraging of allocations based on changes in volatility or variance in the market. For these indices:

$$K_{rb} = \text{Min}(\text{Max } K, \text{Target Variance}/\text{Realized Variance}_{rb-d})$$

where variance is defined as per above.

All other index calculations remain the same.

<sup>12</sup> In accordance with the S&P GSCI approach, the interest rate for these indices is the 91-day U.S. Treasury rate. A 360-day year is assumed for the interest calculations in accordance with U.S. banking practices.

# Risk Control 2.0 Indices

S&P Dow Jones Indices' Risk Control 2.0 Indices are Risk Control indices, where the cash portion of the investment in the standard Risk Control strategy is replaced with a liquid bond index.

The index portfolio consists of two assets, the index for a risky asset  $A$ , with weight  $W$ , and the corresponding bond index  $B$ , with weight of  $(1-W)$ . Weight  $W$  lies between 0 and 100%. There is no shorting or leverage allowed in the strategy.

## Constituent Weighting

The formula to assign weights to the underlying indices is determined by the following:

$$W^2 * \sigma_A^2 + (1-W)^2 * \sigma_B^2 + 2 * W * (1-W) * \rho * \sigma_A * \sigma_B = \sigma_{Target}^2 \quad (64)$$

where:

- $W$  = The weight of the risky asset  $A$
- $\sigma_A$  = The volatility of the risky asset  $A$
- $\sigma_B$  = The volatility of the bond index  $B$
- $\rho$  = The correlation of Index  $A$  and  $B$
- $\sigma_{Target}$  = The target volatility

The calculation of volatility and correlation follows the same procedure and conventions as outlined in the prior section for the standard Risk Control strategy.

The quadratic equation above has two solutions to the weight allocated the index  $A$ :

$$W_1 = (b + \sqrt{b^2 - a * c}) / a$$

$$W_2 = (b - \sqrt{b^2 - a * c}) / a \quad (65)$$

where:

$$a = \sigma_A^2 + \sigma_B^2 - 2 * \rho * \sigma_A * \sigma_B$$

$$b = \sigma_B^2 - \rho \sigma_A \sigma_B$$

$$c = \sigma_B^2 - \sigma_{Target}^2$$

The fallback mechanism for the solutions of weight  $W$ :

1. If none of the solutions in equation (65) above falls between 0 and 100%, then the strategy full back to standard Risk Control, where the maximum leverage is capped at 100%.
2. If both solutions to the equation (65) are valid weights that are greater than 0, then the larger of the two,  $\max(W_1, W_2)$ , becomes the weight of the risky asset  $A$  where the maximum leverage is capped at the level defined by the indices risk control parameters.

The final weights of the underlying assets are determined using the following steps:

**Step 1: Determine the weights under the short term parameters**

- a) Determine the short-term variance for assets A and B using the short term exponential parameter with the same formulae as described in equation (55) under the section Risk Control Indices, with the returns for assets A and B used in determining the short-term variance for assets A and B.
- b) Determine the short-term covariance for assets A and B using similar formulae as described for short-term covariance calculations in equation (55) under the section Risk Control Indices, but replacing the squared equity returns with the product of the risky assets A and B.
- c) Determine the short-term volatility measure for the risky assets A and B from their respective variance measures in the same manner as described in equation (55) under the section Risk Control Indices.
- d) Determine the short-term correlation of A and B from the short-term covariance and the short-term volatility measures.
- e) Determine the possible levels for the weights for A and B using equations (64) and (65) above.

**Step 2: Determine the weights under the long term parameters**

Repeat (a) to (e) in Step 1 above with long-term parameters as described in equation (56) under the section *Risk-Control Indices*.

**Step 3: Determine the final weight W.**

The weight for risky asset A is set equal to the lower of the weight of A as determined in Step 1 and Step 2.

The excess return of the Risk Control 2.0 Indices is calculated as:

$$RiskControl2.0ExcessReturn_t = W * Index_AExcessReturn + (1 - W) * Index_BExcessReturn \quad (66)$$

and the Risk Control 2.0 Index value is:

$$RiskControl2.0IndexValue_t = RiskControl2.0IndexValue_{rb} * (1 + RiskControl2.0ExcessReturn_t) \quad (67)$$

where:

$$RiskControl2.0IndexValue_{rb} = \text{The value of the index at the last rebalancing}$$

Risk Control 2.0 total return indices are calculated in a similar way, where the total return is a weighted sum of total returns of the underlying indices.

Risk Control 2.0 is an extension of standard Risk Control described in detail in the previous section. The parameters used in Risk Control 2.0 follow exactly the way they are calculated in the standard Risk Control methodology.

# Currency and Currency Hedged Indices

A currency-hedged index is designed to represent returns for those global index investment strategies that involve hedging currency risk, but not the underlying constituent risk.<sup>13</sup>

Investors employing a currency-hedged strategy seek to eliminate the risk of currency fluctuations and are willing to sacrifice potential currency gains. By selling foreign exchange forward contracts, global investors are able to lock in current exchange forward rates and manage their currency risk. Profits (losses) from the forward contracts are offset by losses (profits) in the value of the currency, thereby negating exposure to the currency.

## Return Definitions

S&P Dow Jones Indices' standard currency hedged indices are calculated by hedging beginning-of-period balances using rolling one-month forward contracts. The amount hedged is adjusted on a monthly basis.

Returns are defined as follows:

$$\text{Currency Return} = \left( \frac{\text{End Spot Rate}}{\text{Beginning Spot Rate}} \right) - 1$$

$$\text{Unhedged Return} = (1 + \text{Local Total Return}) * (1 + \text{Currency Return}) - 1$$

$$\text{Currency Return on Unhedged Local Total Return} = (\text{Currency Return}) * (1 + \text{Local Total Return})$$

$$\text{Forward Return} = \left( \frac{\text{Beginning one - month Forward Rate}}{\text{Beginning Spot Rate}} \right) - 1$$

$$\text{Hedge Return} = \text{HedgeRatio} * (\text{Forward Return} - \text{Currency Return})$$

$$\text{Hedged Index Return} = \text{Local Total Return} + \text{Currency Return on Unhedged Local Total Return} + \text{Hedge Return}$$

$$\text{Hedged Index Level} = \text{Beginning Hedged Index Level} * (1 + \text{Hedged Index Return})$$

To facilitate index replication, S&P Dow Jones Indices determines the amount of foreign exchange forward contracts sold using an index rebalance reference date.<sup>14</sup> On the index reference date, which occurs on the business day prior to the end of the month, the rebalance forward amounts and currency weights are determined. As a result of the forward amounts and currency weights determination occurring one business day prior to the month end rebalance, an adjustment factor is utilized in the calculation of the hedge return to account for the performance of the S&P Dow Jones Indices Currency-Hedged Index on the last business day of the month. Please refer to the index computation section for further details.

S&P Dow Jones Indices also offers daily currency hedged indices for clients who require benchmarks with more frequent currency hedging. The daily currency hedged indices differ from the standard currency

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<sup>13</sup> By currency risk, we simply mean the risk attributable to the security trading in a currency different from the investor's home currency. This definition does not incorporate risks that exchange rate changes can have on an underlying security's price performance.

<sup>14</sup> Prior to March 1, 2015 S&P Dow Jones Indices' Currency-Hedged Indices utilized the month-end for both index reference and index rebalance date.

hedged indices by adjusting the amount of the forward contracts that mature at the end of month, on a daily basis, according to the performance of the underlying index. This further reduces the currency risk from under-hedging or over-hedging resulting from index movement between two monthly rolling periods.

Details of the formulae used in computing S&P Dow Jones Indices' currency-hedged indices are below.

## The Hedge Ratio

The hedge ratio is simply the proportion of the portfolio's currency exposure that is hedged.

- **Standard Currency-Hedged Index.** In a standard currency-hedged index, we simply wish to eliminate the currency risk of the portfolio. Therefore, the hedge ratio used is 100%.
- **No Hedging.** An investor who expects upside potential for the local currency of the index portfolio versus the home currency, or does not wish to eliminate the currency risk of the portfolio, will use an unhedged index. In this case, the hedge ratio is 0, and the index simply becomes the standard index calculated in the investor's home currency. Such indices are available in major currencies as standard indices for many of S&P Dow Jones Indices' indices.

In contrast to a 100% currency-hedged standard index, which seeks to eliminate currency risk and has passive equity exposure, over- or under-hedged portfolios seek to take active currency risks to varying degrees based on the portfolio manager's view of future currency movements.

- **Over Hedging.** An investor who expects significant upside potential for the home currency versus the local currency of the index portfolio might choose to double the currency exposure. In this case, the hedge ratio will be 200%.
- **Under Hedging.** An investor who expects some upside potential for the local currency of the index portfolio versus the home currency, but wishes to eliminate some of the currency risk, might choose to have half the currency exposure hedged using a 50% hedge ratio.
- **Optimal Hedging.** In order to minimize variability and, therefore, risk in the value of the currency-hedged portfolio, standard variance minimization suggests the following hedge ratio:

$$\text{Hedge Ratio} = \text{COV}(\text{Portfolio Return to Forward Return}) / \text{VAR}(\text{Forward Return})$$

S&P Dow Jones Indices calculates indices with hedge ratios different from 100% as custom indices.

## Calculating a Currency-Hedged Index

Using the returns definitions on prior pages, the Hedged Index Return can be expressed as:

$$\text{Hedged Index Return} = \text{Local Total Return} + \text{Currency Return} * (1 + \text{Local Total Return}) + \text{Hedge Return}$$

Rearranging yields:

$$\text{Hedged Index Return} = (1 + \text{Local Return}) * (1 + \text{Currency Return}) - 1 + \text{Hedge Return} \quad (68)$$

Again, using the returns definitions on prior pages with a hedge ratio of 1 (100%), the expression yields:

$$\text{Hedged Index Return} = \text{Unhedged Index Return} + \text{Hedge Return}$$

$$\text{Hedged Index Return} = \text{Unhedged Index Return} + \text{Forward Return} - \text{Currency Return} \quad (69)$$

This equation is more intuitive since when you do a 100% currency hedge of a portfolio, the investor sacrifices the gains (or losses) on currency in return for gains (or losses) in a forward contract.

From the equation above, we can see that the volatility of the hedged index is a function of the volatility of the unhedged index return, the forward return, and the currency return, and their pair-wise correlations.

These variables will determine whether the hedged index return series' volatility is greater than, equal to, or less than the volatility of the unhedged index return series.

### Currency Hedging Outcomes

The results of a currency-hedged index strategy versus that of an unhedged strategy vary depending upon the movement of the exchange rate between the local currency and home currency of the investor.

S&P Dow Jones Indices' standard currency hedging process involves eliminating currency exposure using a hedge ratio of 1 (100%).

1. The currency-hedged index does not necessarily give a return exactly equal to the return of the index available to local market investor. This is because there are two additional returns – currency return on the local total return and hedge return. These two variables usually add to a non-zero value because the monthly rolling of forward contracts does not result in a perfect hedge. Further, the local total return between two readjustment periods remains unhedged. However, hedging does ensure that these two returns remain fairly close.
2. The results of a currency-hedged index strategy versus that of an unhedged strategy varies depending upon the movement of the exchange rate between the local currency and home currency of the investor. For example, a depreciating euro in 1999 resulted in an unhedged S&P 500 return of 40.0% for European investors, while those European investors who hedged their U.S. dollar exposure experienced a return of 17.3%. Conversely, in 2003 an appreciating euro in 2003 resulted in an unhedged S&P 500 return of 5.1% for European investors, while those European investors who hedged their U.S. dollar exposure experienced a return of 27.3%.

### Index Computation

#### Monthly Return Series (For Monthly Currency Hedged Indices)

$M$  = The month in the calculation, represented as 0, 1, 2, etc..

$SPI\_EH_m$  = The S&P Dow Jones Indices Currency-Hedged Index level at the end of month  $m$

$SPI\_EH_{m-1}$  = The S&P Dow Jones Indices Currency-Hedged Index level at the end of the prior month

$SPI\_EH_{mr-1}$  = The S&P Dow Jones Indices Currency-Hedged Index level at the end of the prior month index reference date. S&P Dow Jones Indices' standard index reference date for hedged indices is one business day prior to the month-end rebalance date.

$SPI\_MAF$  = Monthly Index Adjustment Factor to account for the performance of the S&P Dow Jones Indices Currency-Hedged Index between the index reference and month end rebalance dates. It is calculated as the ratio of the S&P Dow Jones Indices Currency-Hedged Index level on the reference date and the S&P Dow Jones Indices Currency-Hedged Index level at the end of the month.

$$SPI\_MAF = \left( \frac{SPI\_EH_{mr-1}}{SPI\_EH_{m-1}} \right)$$

$SPI\_E_m$  = The S&P Dow Jones Indices Index level, in foreign currency, at the end of month  $m$

$SPI\_E_{m-1}$  = The S&P Dow Jones Indices Index level, in foreign currency, at the end of the prior month

$SPI\_EL_{m-1}$  = The S&P Dow Jones Indices Index level, in local currency, at the end of the prior month,  $m-1$

$HR_m$  = The hedge return (%) over month  $m$

$S_m$  = The spot rate in foreign currency per local currency ( $FC/LC$ ), at the end of month  $m$

$S_{mr}$  = The spot rate in foreign currency per local currency (FC/LC) on the index reference date for month  $m$

$F_m$  = The forward rate in foreign currency per local currency (FC/LC), at the end of month  $m$

For the end of month  $m = 1$ ,

$$S_{PI\_EH_1} = S_{PI\_EH_0} * \left( \frac{S_{PI\_E_1}}{S_{PI\_E_0}} + HR_1 \right)$$

For the end of month  $m$ ,

$$S_{PI\_EH_m} = S_{PI\_EH_{m-1}} * \left( \frac{S_{PI\_E_m}}{S_{PI\_E_{m-1}}} + HR_m \right)$$

The hedge return for monthly currency hedged indices is:

$$HR_m = \left( \frac{F_{m-1}}{S_{mr-1}} - \frac{S_m}{S_{mr-1}} \right) * S_{PI\_MAF}$$

### Daily Return Series (For Monthly Currency Hedged Indices and Daily Currency Hedged Indices)

The daily return series are computed by interpolating between the spot price and the forward price.

For each month  $m$ , there are  $d = 1, 2, 3 \dots D$  calendar days.

$md$  is day  $d$  for month  $m$ ,  $m0$  is the last business day of the month  $m-1$  and  $mr0$  is the index reference day of the month  $m-1$ .

$F_{I_{md}}$  = The interpolated forward rate as of day  $d$  of month  $m$

$AF_{md}$  = The adjustment factor for daily hedged indices as of day  $d$  of month  $m$

$$F_{I_{md}} = S_{md} + \left( \frac{D-d}{D} \right) * (F_{md} - S_{md})$$

$$AF_{md} = \frac{S_{PI\_EL_{md-1}}}{S_{PI\_EL_{m0}}}$$

For the day  $d$  of month  $m$ ,

$$S_{PI\_EH_{md}} = S_{PI\_EH_{m0}} * \left( \frac{S_{PI\_E_{md}}}{S_{PI\_E_{m0}}} + HR_{md} \right)$$

The hedge return for monthly currency hedged indices is:

$$HR_{md} = \left( \frac{F_{m0}}{S_{mr0}} - \frac{F_{I_{md}}}{S_{mr0}} \right) * S_{PI\_MAF}$$

The hedge return for daily currency hedged indices is calculated as follows:

When day  $d$  is the first business day of month  $m$ ,

$$HR_{md} = AF_{mi} * \left( \frac{F_{m0}}{S_{mr0}} - \frac{F_{-I_{mi}}}{S_{mr0}} \right)$$

When day  $d$  is not the first business day of month  $m$ ,

$$HR_{md} = AF_{mi} * \left( \frac{F_{-I_{mi-1}}}{S_{mr0}} - \frac{F_{-I_{mi}}}{S_{mr0}} \right) + HR_{md-1}$$

# Domestic Currency Return Index Calculation

## Background

Domestic Currency Return (DCR) calculations give the same results as divisor based calculations. Moreover, adjustments for corporate actions, additions and deletions of securities and other changes can be done using DCR.

In DCR one calculates the period-to-period percentage change of the index from the weighted percentage change of each security price and then constructs the index levels from the percentage changes. In a divisor based index the process is reversed: the index level is calculated as total market value divided by the divisor and the period-to-period percentage change is calculated from the index levels. Both approaches require an initial base period or divisor value for normalization. Both approaches give the same results. The choice depends on which approach is more convenient for a particular index. When an index of indices or an index with securities in different currencies is constructed the DCR method may be preferred.

In the DCR calculation, we calculate the percentage change in each security price, weight the percentage changes by the security's weight in the index at the start of the period and then combine the weighted price changes to calculate the index price change for the time period. The change in the index is, then, applied to the index level in the previous period to determine the current period index level.

## Equivalence of DCR and Divisor Calculations

The equivalence of the two approaches – DCR and divisor based – can be understood in two ways. First, except for the initial base value of an index, it can be defined by either the index levels or the percentage change from one period to the next. If we defined an index by a time series of index levels (100, 101.2, 103, 105...) we can derive the period to period changes (1.2%, 1.78%, 1.94%...). Given these changes and assuming the index base is a value of 100 allows us to calculate the index levels. Except for the base, the two series are equivalent. DCR calculates the changes; the divisor approach calculates the levels.

The can be shown mathematically:

The divisor calculation approach defines an index as:

$$\frac{\sum_j price_{i,t} * share_j}{divisor} \quad (70)$$

Since the initial divisor is defined by the base value and date of the index, we can replace it with the value of the index market cap at time  $t=0$ :

$$\frac{\sum_j price_{i,t} * share_j}{\sum_j price_{i,0} * share_j} \quad (71)$$

Now we can multiply and divide the term in the summation in the numerator by the price at time  $t=0$  without changing its value.

$$\frac{\sum_i \frac{price_t}{price_0} * price_0 * shares_i}{\sum_i price_{i,0} * shares_i} \quad (72)$$

If we look at the term in the numerator for a single stock in the index (i.e. no summation, as there is only one stock) and rearrange we get:

$$\left( \frac{price_{i,t}}{price_{i,0}} \right) * \frac{price_{i,0} * shares_i}{\sum_i price_{i,0} * share_i} \quad (73)$$

which is equivalent to the relative price performance for each stock multiplied by its weight in the index. When this is combined across all constituent stocks, the result is the price performance for the index.

The DCR approach uses the summation of equation (73) across all the stocks in the index to calculate the daily price performance of the index. Once the daily index performance is calculated, the index level can be updated from the previous day's index level.

#### DCR Calculation

$$Index_t = (Index_{t-1}) * \sum_i \frac{P_{i,t}}{P_{i,t-1}} * weight_{i,t-1} \quad (74)$$

where:

$Index_t$  = Index level at date  $t$

$P_t$  = Security price at the close of date  $t$

$weight_t$  = Security weight in the index at close of date  $t$

and

$$weight_{i,t-1} = \frac{P_{i,t-1} * S_{i,t-1} * FX_{i,t-1}}{\sum_i P_{i,t-1} * S_{i,t-1} * FX_{i,t-1}} \quad (75)$$

where:

$S_{i,t-1}$  = Shares of stock  $i$

$FX_{i,t-1}$  = Exchange rate of stock  $i$  for currency conversion

#### Essential Adjustments

The share count ( $S_{i,t-1}$ ) includes the adjustment for float by multiplying by the investable weight factor ( $IWF$ ) and for index weight by multiplying by the additional weight factor ( $AWF$ ) where necessary. Further, when an adjustment to shares is made due to a secondary offering, share buyback or any other corporate action, this adjustment must be included in  $S_{i,t-1}$  if the adjusted share count takes effect on date  $t$ . A price adjustment due to a corporate action which takes effect on date  $t$  should be reflected in  $P_{i,t-1}$ .

# Fee Indices

S&P Dow Jones Indices calculates fee indices that are meant to alter the index value of a given underlying index according to a fixed rate that is applied on a daily basis. This rate can be either positive or negative, but in most cases the fee index level is lower than the underlying index level.

Fee indices can be calculated in one of two ways. The fee can be applied to the index after the return of the underlying index is calculated, or it can be applied along with the return of the underlying index. The two different calculations are as follows:

Option 1:

$$IndexValue_t = IndexValue_{t-1} * \left( \frac{ParentIndexValue_t}{ParentIndexvalue_{t-1}} \right) * (1 - (Fee / N) * (D_t - D_{t-1}))$$

Option 2:

$$IndexValue_t = IndexValue_{t-1} * \left( 1 + \left( \frac{ParentIndexValue_t}{ParentIndexvalue_{t-1}} - 1 \right) - Fee * (D_t - D_{t-1}) / N \right)$$

where:

$N$  = The number of days in a year

$D_t$  = The current day

# Special Opening Quotation

The special opening quotation (“SOQ”) is calculated using the same methodology as the underlying index except that the price used for each index constituent is the open price at which the security first trades upon the opening of the exchange on a given trading day. SOQ is calculated using only the opening prices from the primary exchange, which occur at various times, of all stocks in the index and may occur at any point during the day. For any stock that has not traded during the regular trading session, the previous day's closing price is used for the SOQ index calculation. SOQ may be higher than the high, lower than the low and different from the open, as the SOQ is a special calculation with a specific set of parameters. The open, high, low and close values are continuous calculations, while the SOQ waits until all stocks in the index are open.

- **U.S. Markets.** If the exchange is unable to provide official opening prices, the official closing prices utilized are determined based on SEC Rule 123C as outlined in the *Unexpected Exchange Closures* chapter of *S&P Dow Jones Indices' Equity Indices Policies and Practices* document.
- **Non-U.S. Markets.** If the exchange is unable to provide official opening prices, the official closing prices are utilized. If the exchange is unable to provide official opening or closing prices, the previous closing price adjusted for corporate actions is used in the calculation of the SOQ.
- For M&A target stocks that are suspended or halted from trading on an exchange but are still in indices, S&P Dow Jones Indices will synthetically derive an SOQ for the suspended security using the deal ratio terms and the opening price of the acquiring company if the acquirer is issuing stock as part of the merger. If the acquirer is paying cash only, the lower of the previous official close price and the cash amount are used in the calculation of the SOQ.

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