Ossiam World Minimum Variance Index

Technical Description

Version 3

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1. Universe

1.1 Reference Universe

Investment universe for the Index is the current composition of the S&P Global 1200 index.

1.2 Liquidity/Capacity Filter

1.2.1 Eligibility filter

Emerging market stocks present in the S&P Global 1200 are excluded from the investment set. Constituents of the S&P Asia 50 and S&P Latin America 40 are therefore excluded from the investment universe.

1.2.2 Capacity filter

This filter sorts out the stocks that have relatively small market capitalization. The constituents selected at the previous step are sorted by their free-float market capitalization, and only the biggest stocks that cumulatively represent ***W***% of the sum of free-float market capitalization of the stocks selected by the eligibility filter are selected.

At each Index review date the selection is updated, applying the buffer rule to induce major stability in the capacity selection. According to this rule, the stocks that are located in between the ***W***% and (***W+U***)% of free-float market capitalization, and that were among the ***W***% biggest stocks at the previous review date, are also selected by the capacity filter.

1.2.2. Dealing with Multiple Listings

If there are multiple shares of the same company present in the selection of the capacity filter, only the most liquid share class or listing for each company is retained (the liquidity is estimated by the Average Daily Volume as will be described below).

1.2.3 Liquidity filter

Only the most liquid stocks from the selection are chosen. For this purpose a liquidity filter is designed, that works in the following way:

a) liquidity is estimated for each stock, using most recent transaction volume data. Average Daily Volume is calculated as a simple average of daily transaction volume series over the past *Tv* days

$$ADV^i = \frac{1}{Tv}\sum_{t=T-Tv+1}^{T} V_t{}^i * P_t^i * C_t$$

where T denotes estimation date, V is volume in number of shares, P is stock price in the stocks' home currency, and C is the relevant exchange rate against USD.

If a stock has more than ***p***% missing volume observations during the liquidity estimation period, it is assigned a zero ADV.

b) the stocks from the selection are ranked by their ADV in descending order,
c) first M stocks having the highest liquidity are selected,
d) buffer rule is applied: the stocks ranked from ***M***+1 to ***M+B*** are selected if and only if they were among the ***M*** most liquid stocks on the previous review date. This buffer helps to avoid some of the turnover associated with the exclusion of the stocks that were among the most liquid and are likely to re-integrate the most liquid selection in the future.

The liquidity filter is applied each time the Index is rebalanced, before calculating new optimized weights.

1.3 Data

The following data is used in the Index construction process:

- F – free-float market capitalization of the stocks at market close on the dates the index revision is calculated,
- P - daily share prices at market close,
- TR – daily share total return price. The total return price is adjusted for corporate actions and dividend payments,
- V - daily transaction volume from the respective stock exchanges,
- C - end-of-day foreign exchange rates, corresponding to 16.00 London time (UTC),
- classification of the stocks in S&P Global 1200 by industrial sector corresponding to the level 1 of the GICS classification, having 10 industries:
    a. Energy
    b. Materials
    c. Industrials
    d. Consumer Discretionary
    e. Consumer Staples
    f. Health Care
    g. Financials
    h. Information Technology
    i. Telecommunication Services
    j. Utilities

2 Portfolio Construction

2.1 General

Index constituents are weighted by an optimization procedure, aimed at minimizing portfolio variance under constraints.

2.2 Return Data

The optimization procedure starts by calculating 3-day period arithmetic price returns:

$$r_t^i = \frac{TR_t^i * C_t}{TR_{t-3}^i * C_{t-3}} - 1$$

where (t-3) denotes the date three business days before day t, TR are total return close prices.

2.3 Variance Estimation Details

2.3.1 Covariance estimation period definition

The covariance is estimated over the period of ***Ts*** days, that are not considered common or partial holidays (see more details in the section 2.8).

2.3.2 Missing data filter

The stocks that have more than ***q***% of missing price observations inside the estimation period will be dropped from the selection. The stocks having an acceptable proportion of missing price observation will be kept in the selection, after filling the missing prices with the "previous" price levels (i.e. the prices from the observation just before the missing ones).**2.3.3. Constant price filter**

If stock prices remain constant across several observations, this will lead to null daily returns. The stocks having a proportion of null daily returns that is bigger than ***Z***% of the total daily return observations inside the estimation period will be dropped from the selection.

2.3.4 Covariance Estimation

For all the stocks admitted to the optimization step a variance-covariance matrix is estimated as follows:

$$\Sigma_T^{i,j} = \frac{1}{Ts - 1} \sum_{t=T-Ts+1}^{T} (r_t^i - \bar{r}^i)(r_t^j - \bar{r}^j)$$

where M is the number of stocks admitted for optimization, $\bar{r}^{\,i}$ denotes average return of the i-th stock.

2.4 Optimization: objective function

The function to be minimized is the variance of the Index portfolio:

$$\sigma_{Ind}^2 = \sum_{i=1}^{M}\sum_{j=1}^{M} w_i \hat{\Sigma}_{\mathrm{ij}} \mathrm{w_j}$$

2.5 Optimization: constraints

The optimization is subject to the following constraints:

e) 100% leverage constraint: $\sum_{i=1}^{M} w_i = 1$

f) long-only constraint: $w_i \geq 0$, for all *i*

g) maximal weight constraint: $w_i \leq \boldsymbol{w_{max}}$

- maximal sector exposure constraint: $w_S \leq \boldsymbol{S_{max}}$

  where $w_S = \sum_{i \in S} w_i$, is net exposure to the sector S.

- diversification target: $\sum_{i=1}^{M} w_i^2 = \frac{1}{\boldsymbol{H}}$

2.6 Optimization: numerical algorithm

The optimization problem is a quadratic constrained minimization problem. It is solved numerically, using an interior-point algorithm. This algorithm calculates an iterative sequence of approximate minimization problems, where inequality constraints are transformed into equality constraints using slack variables. The optimal solution is defined with the help of the following convergence criteria:

h) TolFun - termination tolerance on the function value,
i) TolCon - tolerance on the constraints violations
j) MaxIter - maximal number of iterations allowed

2.7 Rounding issues

Input data to the optimization, as well as all intermediate calculations, are not rounded.

The optimized weights that are smaller than ***wtol*** (i.e. that are essentially zero) are rounded to exact zero. To distribute the cumulative weight of the excluded stocks, an optimization as described in the sections 2.4-2.6 is repeated only for the stocks that remain in the portfolio after the rounding cut-off with an additional minimal weight constraint of $w_i \geq \boldsymbol{wtol}$.

2.8 Estimation period definition

For the covariance estimation we only consider the days that are not common or partial holidays. A common holiday is a date when all the stocks in the base index are not traded. A partial holiday is a date where one of the major stocks exchanges was closed, or when several exchanges are simultaneously closed. The exact list of holidays is given in the appendix.

The same restriction is applied to the liquidity (ADV) estimation.

3 Index Calculation

3.1 Base date

At the Base Date the Index level is equal to 100.

3.2 Trading Days and Holidays

The Index is calculated and disseminated according to the US equity market schedule.

3.3 Calculation Frequency

The Index is calculated in real time every 15 seconds between 8.00 a.m. and 5.15 p.m. EST. The closing value is calculated at 5:30 p.m. Eastern time.

3.4 Currency

The Index is calculated in USD.

3.5 Total Return

Price return and Net Total return versions of the Index are calculated.

3.6 Rebalancing Schedule

Rebalancing takes place on a semiannually basis, on third Friday of March and September. The new Index composition becomes effective at the opening of the US markets on the date after the rebalancing date.

The optimal weights are calculated after the market close on the reference date that is *K* business days prior to the rebalancing date.

3.7 Index Value Formula

Between two rebalancing dates the Index is calculated as follows:

$$Index_t = \frac{\sum_{i=1}^{M} P_t^i q_{Tprev}^i C_t}{D_t}$$

We define a weighting factor of a stock as a ratio of the stock's weight to the stock's price

$$q_t^i = \frac{w_t^i}{P_t^i C_t}$$

At each rebalancing date T each stock is assigned a weighting factor that is based on its target weight and the day-T close price. The divisor D transforms the value of the hypothetic index portfolio into index level.

3.8 Treatment of corporate actions and changes in the investment universe

Between two rebalancing dates the maintenance of the Index constituents is based on the following principle: all share and price adjustments that do not alter the membership of stocks in the investment universe or their risk characteristics do not lead to changes in the Index value or composition. Below we detail the maintenance rules for the most common corporate actions. For all

the cases not explicitly mentioned in this document the maintenance is made according to the general practices for the S&P index family.

3.8.1 Early exits

If a company that is currently present in the Index is excluded from the investment universe (represented by the S&P Global 1200 Index) between two subsequent rebalancing dates, it is immediately excluded from the Index portfolio and its weight is distributed pro-rata among the remaining stocks

$$w_i \rightarrow \frac{w_i * 100\%}{1 - w_{exit}}$$

3.8.2 Regular Dividends

Dividends received on a stock present in the Index are reinvested in the Index on a net basis. Net dividend is a gross dividend minus withholding tax. Amounts of unadjusted gross cash dividends and withholding tax amounts are calculated using S&P assumptions. The adjustment corresponding to the reinvestment of net dividend is done on the date after the ex-dividend date at the market opening.

Consequently, dividend payment and reinvestment does not change the weights of the Index components, but results in an adjustment of the Index level as follows

$$Index_t \rightarrow Index_t(1 + \frac{Div_t}{Index_t})$$

where Div_t is the total net dividend in USD received on the day t and divided by the current index divisor

$$Div_t = \frac{\sum d_i N_i}{D}$$

N_i is the number of shares of the *i*-th company, and d_i is the per-share net dividend expressed in USD. If the company pays dividends in a currency different from USD, the amount of the dividend in converted to USD using the foreign exchange rate at the equity market close.

3.8.3 Special Dividends

Special dividends are those dividends that are outside the normal payment pattern established historically by the issuing corporation. For the detailed definition we refer to the S&P Index Methodology Guide. If a company present in the Index pays a special dividend, the price of the company is adjusted downwards by the amount of the dividend after the close of trading on the day before the ex-dividend date

$$Adjusted\ Price = Close\ Price - d_i$$

Accordingly, the Index divisor is adjusted to compensate for the price drop

$$D \to D * \frac{D * Index - N_i * d_i}{D * Index}$$

This results in a decrease of the company's weight in the Index, since its weighting factor remains unchanged.

3.8.4 Spin-offs

If a company present in the index has a spin-off, the spun-off company is not added to the Index. There are two possible scenarios:

1) if the spun-off company has a price, there is a corresponding price drop in the spin-off company, that affects its weight. Then the spun-off company shares are sold and the proceedings are reinvested in the index.
2) If the spun-off company has no price, it is held in the index with a price 0, and there is no adjustment to the price/weight of the spin-off company. When the first trading price for spun-off is available, the scenario 1 is applied.

3.8.5 Merger & Acquisition

We denote companies by the capital letters A, B, C,…

1) Merger: A+B = C
   a. if A and B are in the Index: $w_C = \min(w_A + w_B, w_{max})$
   b. if A is in the Index, and B is not: $w_C = w_A$
2) Acquisition: A+B =A
   a. If A and B are in the Index: $w_A = \min(w_A + w_B, w_{max})$
   b. If only A is in the Index: $w_A = w_A$
   c. If only B is in the Index: $w_B = 0,$

the acquired stock is eliminated from the Index and the proceedings are reinvested pro-rata in the remaining stocks.

The maximal weight limit w_{max} is the same that is used for portfolio optimization constraint.

3.8.6 Share Conversion: A-> B

If a company converts its shares from one class to another, and the class A that was present in the portfolio is converted to a class B that was not, then we keep the converted shares in the Index until the next rebalancing. At the next rebalancing the new share class is considered a new entity, and does not inherit the historical price/volume data of the suppressed share class. This new share class is considered for the inclusion in the Index on the next rebalancing date only if it is a component of the investment universe.

3.8.7 Summary Table

EVENT TYPE	IMPACT ON THE INDEX
Company Addition to the investment universe	No change
Company Deletion from the investment universe	If the deleted company is in the Index, it is dropped and its weight is reinvested pro-rata in the remaining stocks
Price Adjustments	Price of the stock and number of shares are adjusted to reflect the corporate action, so the weight of the stock in the index stays the same
Share Issuance/ Buy Back	No change
Rights Offering	Price adjusts down and number of shares adjusts up so the weight of the stock stays the same
Spin-off	The spun-off company is deleted from the Index and its weight is reinvested pro-rata in the remaining stocks
Change of Stock Float Factor	No change
Merger and Acquisition	The acquiring company is given a weight that is a minimum between the sum of the old weights of the acquiring company and the target company in the Index and the maximum weight limit w_{max}. If two companies merge, the weight for the resulting company is a minimum of the sum of the weights of the merging companies and the maximum weight limit w_{max}
Special Dividend	The price of the stock is adjusted down by the amount of the dividend, with no adjustment to the number of shares, but with the downward adjustment in the Index divisor. The weight of the stock making the special dividend payment goes down
Regular Dividend	The net dividend is reinvested in the total return index only, with no adjustment made in the divisor and in the weights

4 Parameters

W	=	85%	market capitalization threshold
U	=	3%	market capitalization buffer
p	=	20%	maximum share of missing values inside liquidity estimation period accepted
Tv	=	125 days	liquidity estimation period
M	=	400	number of the most liquid stocks selected by liquidity filter
B	=	50	liquidity buffer
Ts	=	500	number of 3-days periods returns for covariance estimation
q	=	10%	maximal share of missing values inside the covariance estimation period
Z	=	40%	maximal proportion of zero return observations inside covariance estimation period
wmax	=	3.5%	maximal weight
Smax	=	20%	upper bound for single sector exposure
Sect. Classif	=	GICS	Sector classification
H	=	80	inverse diversification target
TolFun	=	10^{-12}	termination tolerance on the objective function value
TolCon	=	10^{-8}	tolerance on constraints violation
MaxIter	=	10^{12}	maximal number of iterations
wtol	=	10^{-3}	significance threshold for weights

Base Date	=	2012/03/16	
Base Value	=	100	
K	=	5 days	gap between weight calculation date and rebalancing date