S&P Dow Jones Indices

A Division of S&P Global

Fixed Income Index Mathematics
Methodology

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Introduction

Highlights

This document covers the mathematics of fixed income index calculations. To understand and successfully use indices for investment analyses, it is important to know how they are calculated, how coupon payments are handled, and how other adjustments are made to the index. While actual index calculations are done almost entirely by computer, utilizing a wide range of programs, algorithms and routines, the underlying math is fairly straightforward. For details on a specific index, please refer to that index's methodology document available at *www.spdji.com*.

Different Varieties of Indices

A majority of S&P Dow Jones fixed income indices are market value weighted, where each bond's weight in the index is proportional to its market value. Sometimes an index has capping requirements which set the target weights for the index securities. An additional weight factor (AWF) is used to make market value adjustments to the index securities in order to satisfy the capping rules.

Index Calculations

Daily Index Values

Index values are calculated each day by applying the current day's index return to the previous day's index value, as follows:

$$TRIV_t = TRIV_{t-1} * (1 + IndexTR_t) \quad (1)$$

$$PRIV_t = PRIV_{t-1} * (1 + IndexPR_t \quad (2)$$

$$IRIV_t = IRIV_{t-1} * (1 + IndexIR_t) \quad (3)$$

where:

$TRIV_t$	=	Total return index value on day *t*.
$PRIV_t$	=	Price return index value on day *t*.
$IRIV_t$	=	Interest return index value on day *t*.
$IndexTR_t$	=	Index total return on day *t*.
$IndexPR_t$	=	Index price return on day *t*.
$IndexIR_t$	=	Index interest return on day *t*.

Daily Index Returns

The individual index security returns are aggregated to calculate returns for the index. Specifically, on a given day, the total return, interest return and price return for the index are equal to a weighted average of the returns of the securities that constitute the index. The weight of each index security return is equal to the relative weight of that security in the index as of the previous calendar day (adjusted for principal pre-payments, etc.). Each cash security has a foreign exchange return related to the index currency as its price return and zero as its interest return. The formulae are as follows:

$$IndexTR_t = \sum_i SecurityWeight_{i,t-1} * tr_{i,t} \quad (4)$$

$$IndexPR_t = \sum_i SecurityWeight_{i,t-1} * pr_{i,t} \quad (5)$$

$$IndexIR_t = \sum_i SecurityWeight_{i,t-1} * ir_{i,t} \quad (6)$$

where:

$IndexTR_t$	=	Index total return on day *t*.
$IndexPR_t$	=	Index price return on day *t*.
$IndexIR_t$	=	Index interest return on day *t*.
$tr_{i,t}$	=	Total return of index security *i* on day *t* ($ir_{i,t} + pr_{i,t}$).
$pr_{i,t}$	=	Price return of index security *i* on day *t*.
$ir_{i,t}$	=	Interest return of index security *i* on day *t*.
$SecurityWeight_{i,t-1}$	=	Adjusted market value weight of index security *i* at the close of day *t-1*.

Total Return

The total return, *TR,* for a given security on day *t* is the sum of the market price return and the interest return on day *t*:

$$TR_t = IR_t + PR_t \quad (7)$$

where:

IR_t = Interest return on day *t*.

PR_t = Price return on day *t*.

Price return measures the return due to the change in the market price of the security. Interest return (or coupon return) includes the return due to the interest earned on that security. In the case of zero coupon bonds, the accretion in price due to interest return is reported as price return.

Interest Return

The formula for the interest return on an individual index security on day *t* is as follows:

$$IR_t = (AI_t - AI_{t-1} + \text{Cpn}_t) / DirtyPrice_{t-1} \quad (8)$$

where:

IR_t = Daily interest return for the security on day *t*.

AI_t = Accrued interest, up to and including day *t*.

$DirtyPrice_{t-1}$ = Dirty price of the security on day *t-1*.[1]

Cpn_t = Coupon payment* on day *t*.

**For securities trading ex-dividend, the coupon is recognized on ex-dividend date.*

Interest Return (Loans)

In the following formula, *PAR* should be treated as (*AWF*PAR*). The formula for the interest return on an individual index loan on day *t* is as follows:

$$IR_t = \frac{(PAR_t * R_t) / 360}{MV_{Beg}} \quad (9)$$

where:

IR_t = Interest return on day *t*.

PAR_t = Par amount of the index loan as of the last weekly rebalancing, adjusted for principal pre-payments, etc., up to and including day *t*.

R_t = Interest rate on day *t*.

MV_{Beg} = Market value, at the beginning of day *t*.

Index Interest Rate

The index interest rate is determined by the weighted average spread to LIBOR/EURIBOR.

[1] The dirty price of a security is defined as the sum of the market quoted price and the interest deemed to be earned on that security, but not yet paid to the investor. The clean price is the market quoted price without accrued interest.

Price Return

The formula for the price return for an index security on day *t* is as follows:

$$PR_t = (CleanPrice_t - CleanPrice_{t-1}) / DirtyPrice_{t-1} \qquad (10)$$

where:

PR_t = Price return for the security on day *t*.

$CleanPrice_t$ = Market quoted price for the security without accrued interest on day *t*.

$DirtyPrice_{t-1}$ = Market quoted price for the security with accrued interest on day *t-1*.

Price Return (Loans)

The formula for the price return for an index loan on day *t* is as follows:

$$PR_t = \frac{PAR_t * \left(\frac{P_t - P_{t-1}}{100}\right) + Prin_t * \frac{RP - P_{t-1}}{100}}{MV_{Beg}} \qquad (11)$$

where:

PR_t = Price return on day *t*.

PAR_t = Par amount of the index loan as of the last weekly rebalancing, adjusted for principal pre-payments, etc., up to and including day *t*.

P_t = Loan price on day *t*.

P_{t-1} = Loan price on the previous day.

$Prin_t$ = Principal pre-payments, etc., on day *t*.

MV_{Beg} = Market value, beginning of day *t*.

RP = Redemption price.

Note that the formula for the Price Return (11) itself has two components. The first term, in the numerator on the left side, represents the unrealized return due to any change in the price, while the second term (on the right) represents the realized return due to receiving a principal repayment at the Redemption Price (which could differ from par) rather than at the current end of day price.

Unhedged Return

Total unhedged return measures the return by converting the local currency return of the underlying securities to the index currency. The formula for the unhedged daily return for a security is composed of its local currency return and the foreign currency return.

$$TR_{UH,t} = (1 + R_{L,t}) * \left(1 + \frac{FX_t - FX_{t-1}}{FX_{t-1}}\right) - 1 \qquad (12)$$

where:

$TR_{UH,t}$ = Total unhedged return on day *t*.

$R_{L,t}$ = Total local currency return on day *t*.

FX_t = Foreign exchange spot rate on day *t*.

FX_{t-1} = Foreign exchange spot rate on day *t-1*.

Hedged Return

Hedged return measures the return by hedging currency risk through a one-month forward currency contract. The formula for the hedged return for an index is shown as follows:

$$HedgedMTD_t = H_t + H_t * \left(\frac{S_0}{F_{0,30}} - \frac{S_t}{F_{t,30-t}}\right) + (1 + BaseMTD_t - H_t) * \frac{S_0}{S_t} \quad (13)$$

$$TR_{H,t} = \left(\frac{HedgedMTD_t}{HedgedMTD_{t-1}}\right) - 1 \quad (14)$$

where:

$TR_{H,t}$	= Total hedged return on day t
$HedgedMTD_t$	= Month to date hedged return on day t
$BaseMTD_t$	= Month to date return of the local currency index on day t
S_t	= Foreign exchange spot rate on day t
$F_{0,30}$	= Forward rate on the rebalancing date 0 with 30 days remaining in the contract, assuming a 30/360 day count convention.
$F_{t,30-t}$	= Interpolated forward rate on day t with 30-t days remaining in the contract
H_t	= Hedge size on day t

Hedge Size

The hedge size is determined by using the index weighted yield of the pro forma index on the rebalancing day. Assuming the yield is an annual return proxy, the size of the monthly hedge is a projection of the monthly return of the local currency index. The hedging size used for a MTD calculation on day t is shown by H_t.

$$HedgeSize = 1 + \left(\frac{yield}{2}\right)^{1/6} \quad (15)$$

$$H_t = HedgeSize^{t/30} \quad (16)$$

Market Value

The market value of the security as represented in the index as of the close on day *t* is calculated as follows:

$$MV_t = PAR_t * \frac{(P_t + AI_t)}{100} * FX_t \quad (17)$$

The adjusted market value is applied as follows:

$$AMV_t = AWF * MV_t \quad (18)$$

where:

MV_t	= Market value of the index security on day *t*.
AMV_t	= Adjusted market value of the index security on day *t*.
PAR_t	= Par amount of the index security as of the last rebalancing, adjusted for principal pre-payments, etc., up to and including day *t*.
P_t	= Clean price of the index security on day *t*.
AI_t[2]	= Accrued interest on the index security, up to and including day *t*.

[2] For loans, AI_t is calculated on a 360-day basis. Accrued interest is reduced to zero every 90 days after a loan enters the index.

FX_t	=	Foreign exchange rate used to convert to the target currency on day *t*.
AWF	=	Additional weight factor.

If the valuation date is not a trading day, the market value is based on the price as of the immediate prior trading day, plus interest accrued to the valuation date. On the cash payout day, the accrued interest is set to the coupon cash value.

Additional Weight Factor

The Additional Weight Factor (AWF) is used to tilt the original market value weight of a security within the index. It is calculated on the rebalancing reference date and is implemented on the rebalancing effective date. The AWF remains static until the subsequent rebalancing.

The AWF for all market value-weighted indices is equal to 1. For target weighted indices, the AWF is calculated as follows:

$$AWF_i = TW_i * \frac{\sum_1^N MV_i}{MV_i} \quad (19)$$

where:

AWF_i	=	Additional weight factor of the index security *i*.
TW_i	=	Target weight for the index security *i*.
MV_i	=	Market value of the index security *i* in the underlying index.
$\sum_1^N MV_i$	=	Aggregate market value of all *N* securities in the underlying index.

In equal-weight indices $TW = 1/N$, where N is the number of securities included in the index.

Weight

The relative weight of an index security *i* is defined as the adjusted market value of that security expressed as a percentage of the aggregate adjusted market value of all the securities in the index portfolio, as follows:

$$weight_i = \frac{AMV_i}{\sum_i AMV_i} \quad (20)$$

Yield to Maturity Adjustment for Inflation-linked Securities[3]

To calculate the yield to maturity adjustment, one has to project the inflation rate for future cash flows of the bond. The projected inflation rate is an approximation of future inflation and is calculated using the following formula:

$$\text{Projected inflation Rate}_t = 100 * \left(\frac{CPI_t}{CPI_{t-1year}} - 1\right) \quad (21)$$

where:

CPI_t	=	The reference national CPI for day *t* is calculated through a linear interpolation of the two most recent CPI levels available as of the start of the month.
$CPI_{t-1year}$	=	The reference national CPI as of one year before the valuation date *t* with the same interpolation as CPI_t.

[3] Nominal yield is applied to inflation/index-linked securities only.

Using the projected inflation rate, the current index ratio and the nominal cash flows, we generate the expected inflated cash flows, which are then used to calculate the nominal yield.

Hedge Adjusted Yield

The hedge adjusted yield represents the yield of an index after accounting for the cost of hedging foreign currency exposure in the forward market. At each rebalance, a projected market value using the pro-forma index yield is hedged using the forward rate. The hedge adjusted yield accounts for the premium or discount of the forward rate and applies it to the yield used to project the future market value as follows:

$$\textit{Hedge adjusted yield}_i = \text{Yield}_{i,t} + \left(\frac{FF_{0,30}}{FX_t} - 1\right) * 12 \quad (22)$$

Where:

Yield $_{i,t}$	=	The index weighted yield found in the rebalancing day's pro-forma universe
$FF_{0,30}$	=	Forward rate on the last rebalancing date 0 with 30 days remaining in the contract, assuming a 30/360 day count convention.
FX_t	=	Foreign exchange spot rate on day *t*.

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