The Value of Research: Skill, Capacity, and Opportunity

“When you can measure what you are speaking about, and express it in numbers, you know something about it.”

Lord Kelvin, 1883

EXECUTIVE SUMMARY

How much should a portfolio manager be willing to pay for research? The question is of importance to any manager, but has become particularly pertinent since newly imposed European rules require that the costs of investment research—previously offered by many investment banks as an in-kind consideration in return for brokerage business—be unbundled from trading.

Unfortunately, attempts to determine a fair value for research in the most general circumstances are doomed to fail. Even if we only consider direct recommendations to buy or sell certain securities, the value of such recommendations to a portfolio manager will vary according to the absolute size of positions taken in response. Instead, we provide a framework for estimating relative research values across markets and constituents, under certain stylized (but reasonable) assumptions.

Exhibit 1 provides a summary of our main result—comparing the putative value of recommendations in selected markets, expressed as a multiple of the equivalent measure applied to stock-based recommendations within the S&P 500®.

Exhibit 1: Average “Value of Research” in Various Markets and Segments

<table>
<thead>
<tr>
<th>Market/Segment</th>
<th>Value of Research</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500 Financials</td>
<td>0.66</td>
</tr>
<tr>
<td>S&amp;P Europe 350</td>
<td>0.55</td>
</tr>
<tr>
<td>S&amp;P/TOPIX 150</td>
<td>0.3</td>
</tr>
<tr>
<td>S&amp;P 500 Materials</td>
<td>0.27</td>
</tr>
<tr>
<td>S&amp;P Emerging BMI</td>
<td>0.23</td>
</tr>
<tr>
<td>S&amp;P MidCap 400</td>
<td>0.05</td>
</tr>
<tr>
<td>S&amp;P Developed Ex-U.S. BMI Countries</td>
<td>6.79</td>
</tr>
<tr>
<td>S&amp;P 500 Sectors</td>
<td>7.49</td>
</tr>
<tr>
<td>S&amp;P 500 Energy</td>
<td>1.21</td>
</tr>
<tr>
<td>S&amp;P 500 Information Technology</td>
<td>1.58</td>
</tr>
<tr>
<td>S&amp;P Emerging BMI Countries</td>
<td>3.07</td>
</tr>
<tr>
<td>S&amp;P MidCap 400</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Source: S&P Dow Jones Indices LLC. Data based on monthly averages from October 2008 to September 2018. Chart is provided for illustrative purposes. Past performance is no guarantee of future results.
INTRODUCTION: THE IMPACT OF “UNBUNDLING” RESEARCH COSTS

The Markets in Financial Instruments Directive (MiFID) II is an updated version of a regulation that has been in force throughout the European Union (EU) since November 2007.¹ The update came into effect on January 3, 2018, and seeks “to reform market structures, bring more transparency to the trading of financial instruments, and strengthen investor protection.”²

For our purposes, the relevant regulatory change is that execution costs and charges must be separated, or “unbundled,” from the cost of research, and that investment managers must either absorb research costs or explicitly pass them on to their clients under pre-agreed terms.³ Since investment managers were formerly allowed to pay for research by the allocation of client trading commissions, MiFID II has the potential to produce major changes in the economics of research sales.

While these rules are of most immediate concern to institutions operating in the EU, MiFID II has potential global implications: the updated directive applies to all firms that conduct business in Europe, and many expect the legislation to be extended to other regions.⁴,⁵

From a practical perspective, MiFID II requires managers to set research budgets and to decide where to spend them. Obviously, the size of a particular research budget will depend on idiosyncratic factors, such as a firm’s assets under management. But when it comes to allocating resources, the relative value of research is likely to be comparable—if I find one analyst’s recommendations to be worth double those of other analysts, it is reasonable to hypothesize that these recommendations would also prove to be twice as valuable to anyone else.

This paper argues that the relative value of research is driven by a combination of three things: the information content of the research, the dispersion within the market where recommendations are made and implemented, and the capacity of each market to allow for active positions of varying sizes. While we do not claim to offer a universally applicable framework for setting research budgets, we hope to offer a practical and useful way to think about the value of signals for markets of varying size, concentration, and risk levels.

¹ For more details on the original directive, as well as the updated form, please see the European Securities and Markets Authority (ESMA) webpage.
³ Ibid.
⁵ “MiFID II is driving global standards for research unbundling, says RSRCHXchange,” Institutional Asset Manager, June 12, 2018.
THE UNKNOWN VARIABLE OF “INFORMATION CONTENT”

Some research reports provide explicit recommendations and quantitative predictions—“buy,” “sell,” or “hold,” price targets, future earnings estimates, and so on. Other reports offer more implicit or conditional recommendations, which may be accompanied by an associated degree of conviction. The success or failure of such recommendations can be measured in a multitude of ways, the simplest of which is a “hit rate” (the percentage of times that following a set of recommendations would have improved performance).

More generally, the term “information coefficient” (IC) typically represents a correlation between predictions and outcomes. For example, we might measure the correlation between future percentage changes implied by a series of price targets offered at the start of a period to the actual percentage changes in price measured at the end of the period.

It would be convenient if one could ascertain the IC of research in advance. However, measures of skill tend to be historically biased and hard to measure at the benchmark level—except to the extent that S&P Indices Versus Active (SPIVA®) reports demonstrate the difficulty of maintaining a high batting average. To avoid paralysis by such uncertainty, for the following sections, we shall assume that the IC is a small positive number that is known in advance by the manager.

If the IC is a given, the next important factor is the magnitude of excess performance that predictive recommendations can generate.

DISPERSION AND MEASURING OPPORTUNITY FOR SKILL

Suppose one could purchase a subscription to signals recommending purchases or sales of securities in two different markets (or time periods), both with a similar degree of accuracy. In which market should a recommendation be preferred?

---

6 The terms “information content” and “information coefficient” are sometimes used interchangeably. We shall use the former in the general sense of accurate predictions, reserving the latter term for the case where such accuracy is measured by a correlation statistic.

7 SPIVA Scorecards illustrate the relative rarity with which active funds outperform their benchmarks and can be found at https://spindices.com/SPIVA.

8 We may as well assume the IC is positive, since a negative IC, if known in advance, offers a contrarian indicator.
A report with perfect foresight will be of little value if the difference in performance between the relative winners and losers is minimal. We must therefore consider the dispersion of the market in which the recommendation is being made.\footnote{For more details on how dispersion can help to measure market opportunity, please see Edwards, Tim and Craig J. Lazzara, "Dispersion: Measuring Market Opportunity," S&P Dow Jones Indices LLC, December 2013.}

Clearly, even a report with perfect foresight will be of little value if the difference in performance between the relative winners and losers is minimal. We must therefore consider the dispersion of the market in which the recommendation is being made.\footnote{For more details on how dispersion can help to measure market opportunity, please see Edwards, Tim and Craig J. Lazzara, "Dispersion: Measuring Market Opportunity," S&P Dow Jones Indices LLC, December 2013.}

We must consider the dispersion of the market in which the recommendation is being made.

In order to illustrate the time-varying levels of dispersion, Exhibit 2 shows monthly dispersion levels in the S&P 500 since January 1990—calculated as weighted, annualized standard deviations among the monthly returns of S&P 500 constituents. Absent any other changes, correct recommendations to buy or sell constituents would have been better rewarded in high-dispersion periods, such as 1999 or 2000, than in less-disperse periods, such as 2013 or 2014.

**Exhibit 2: S&P 500 Dispersion Peaked in 1999-2001**

![S&P 500 Dispersion Chart](chart.png)

Source: S&P Dow Jones Indices LLC. Based on annualized monthly data from January 1990 to September 2018. Chart is provided for illustrative purposes. Past performance is no guarantee of future results.

Offering a cross-market perspective, Exhibit 3 shows average (monthly annualized) dispersion figures for a variety of markets over the 10-year period ending in September 2018. Note that we allow for markets to be divided with different levels of granularity. For example, we can measure the dispersion among countries and sectors, as well as among stocks, for indices such as the S&P Emerging BMI.
We can measure the dispersion among countries and sectors, as well as among stocks.

A basket of the better-performing stocks in the S&P Frontier BMI would likely have delivered considerably higher outperformance...

...than a basket of the better-performing single-sector indices selected from the S&P Developed Ex-U.S. BMI.

For market participants of sufficiently small size, dispersion alone may be an appropriate guide to the value of research.

Exhibit 3: Average Dispersion of Various Markets and Segments

Source: S&P Dow Jones Indices LLC. Data based on average annualized monthly dispersion figures from October 2008 to September 2018. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance. Past performance is no guarantee of future results.

Exhibit 3 provides a measure of outperformance opportunity. A basket of the better-performing stocks in the S&P Frontier BMI would likely have delivered considerably higher outperformance than a basket of the better-performing single-sector indices selected from the S&P Developed Ex-U.S. BMI, for example. Similarly, the returns from making successful bets among U.S. small caps, as represented by the S&P SmallCap 600®, would appear to be higher than making similarly successful bets among S&P 500 sectors; the dispersion among the former was more than three times higher.

For market participants of sufficiently small size, dispersion alone may be an appropriate guide to the value of research. Such an investor could form a view of the relative value of research on various markets simply by rescaling the data of Exhibit 3. For example, recommendations pertaining to S&P SmallCap 600 members would be over 50% more valuable than equally accurate recommendations for S&P 500 constituents. Recommendations of S&P 500 sectors would be worth less than one-half the value of equally accurate recommendations of S&P 500 stocks.
For most investors and asset owners, however, research is not automatically worth more when dispersion is higher, due to the problem of varying capacities. If an investor has a single U.S. dollar to deploy, the potential outperformance from picking wisely among S&P SmallCap 600 stocks might be higher than picking among S&P 500 sectors, but if managing a multi-billion-dollar portfolio, the investor will be more limited in the extent to which active positions in small-cap stocks can be established.

We can adjust dispersion to incorporate these capacity issues, and the degree of portfolio “active share” they imply, by placing both in the context of an existing measure of the value of skill to active managers—the so-called “Fundamental Law of Active Management.”

**DISPERSION AND THE FUNDAMENTAL LAW OF ACTIVE MANAGEMENT**

Many readers will be familiar with the Fundamental Law of Active Management, first outlined in the late 1980s.\(^\text{10}\) It states that an active manager’s information ratio (\(IR\)) is positively related to his skill in identifying opportunities for excess returns (\(IC\)), scaled by the number of independent bets that manager has implemented (breadth or \(BR\)). The functional form is given by the following equation:

\[
IR = \sqrt{BR} \times IC
\]

As a heuristic, the equation offers a guiding principle for interpreting the impact of skill (in our case, the IC of research reports) on risk-adjusted active returns. The Fundamental Law offers the considerable appeal of simplicity and provides a wholly intuitive interpretation of the investment management process.\(^\text{11}\)

A simplified example can illustrate the potential roles of IC, dispersion, concentration, and active share in determining the return and information ratio of active portfolios.\(^\text{12}\) We assume that excess returns are random and identically distributed across all stocks, and that an analyst’s recommendation to buy or sell is implemented by overweighting or underweighting at the limit of some predetermined capacity in each position.

After these simplifications, we may write the expected active return and information ratio of a portfolio implementing recommendations as follows:

---


\(^\text{11}\) In what might be a potentially flattering comparison between the speed of light and the skill of active managers, if skill is represented as \(c\) and breadth by \(M\), we can square both sides and define some \(E\), a term related to the value added by a manager, which is equal to \(Mc^2\). This formulation was part of Grinold’s original explanation of the law, and may account for some of its subsequent fame.

\(^\text{12}\) See Appendix A for full details of our simplification, along with a limited amount of algebra.
The Value of Research: Skill, Capacity, and Opportunity

**Expected Active Return**

\[ \text{Expected Active Return} = IC \times D \times TAS \]

**Expected Information Ratio**

\[ \text{Expected Information Ratio} = IC \times \sqrt{M} \times TAS \]

Where

- \( IC \) is the information coefficient, defined as the correlation between the predicted sign of excess returns and the realized excess returns in recommended constituents,
- \( D \) is the capacity-weighted dispersion of the constituents,
- \( TAS = \sum w_i \) is the total active share, which is equal to the sum of the absolute values of the active weights \( w_i \),
- \( M = \left( 1 / \sum w_i^2 \right) \) is a measure of the “effective number” of positions in the active portfolio, which is equal to the reciprocal of the sum of the squared active weights.\(^{13}\)

It is worth emphasizing that the active return equation offers more than a formalization of obvious truths; it identifies the relative role of skill, active share, and dispersion in active returns.\(^{13}\)

INTEGRATING RESEARCH COSTS

How much should a manager be willing to pay to receive recommendations to overweight or underweight? The addition of fixed research costs makes no difference to return volatility in our simple example,\(^{14}\) but the cost per recommendation (\( C \)), expressed as a percentage of portfolio value, must be justified by an incremental expected return.

In our model of simple, directional recommendations to overweight or underweight, the expense in obtaining recommendations would be justified if it satisfies the following equation.

\[ C < IC \times D \times \frac{TAS}{N} \]

Here \( N \) is the number of research recommendations required to determine the active positions. The equation reflects the real world observation that, all else being equal, larger absolute research costs are justifiable.

---

\(^{13}\) We prefer \( TAS \) or total active share, since convention limits the term “active share” to the sum of the absolute values of the active weights divided by 2. See Appendix B for further remarks on the “effective number,” \( M \), and its relation to measures of concentration.

\(^{14}\) By assuming that excess returns are independent and identically distributed, we avoid the much more difficult problem of portfolio construction where returns might be correlated.
when their information content is higher, in markets where there is higher dispersion, for active strategies with higher total active share, or (since $C$ is expressed as a percentage of the notional portfolio size) for larger portfolios.

**INCORPORATING CAPACITY**

Exhibit 3 showed that there was typically much higher dispersion in the monthly performance of the S&P Frontier BMI’s constituents than among—for example—those of the S&P 500. However, frontier market stocks are on average much smaller than S&P 500 members; the former had an (index-weighted) average capitalization of $2.7 billion versus $218.9 billion for S&P 500 constituents (a ratio of more than 80 to 1). In frontier markets, the expected percentage reward from successful active bets might be greater, but the size of an investor’s position might have to be smaller.

How should we account for capacity in our analysis? It’s an elusive concept, not least because to a large degree, it depends on each investor’s wealth and constraints. A $100 million hedge fund, in other words, would have fewer capacity constraints than a $100 billion sovereign wealth fund. Despite the individuality of investors’ capacity constraints, there may be a common, or generally applicable, element, and that element is market capitalization. There is, in other words, more capacity to maintain overweights or underweights in Apple, the largest stock in the S&P 500, with its $1 trillion market capitalization, than there is to do the same in Quanta Services, currently the smallest stock in the S&P 500, with a capitalization of $5 billion.

We might reasonably suppose that an investor’s ability to take active positions in each index constituent is proportional to that constituent’s market capitalization. This assumption does not require the active portfolio to be particularly large to be realistic; real-world variables such as trade impact costs and position limits often scale proportionally to market capitalization, at least in approximate terms. The assumption allows us to examine the potential value of recommendations in various markets and market segments as a function of the dispersion and average capitalization of an index’s constituents.

---

15 Constituent size data in this section reflect free-float market capitalizations as of Sept. 30, 2018. Source: S&P Dow Jones Indices LLC.

16 To be precise, float-adjusted market capitalization.

17 Importantly, a long-only manager’s ability to take negative bets is completely constrained by the weight of each stock in his benchmark.
Exhibit 4 plots a comparison between the average monthly dispersion and the average capitalization of constituents (or segments) for the same set of markets as Exhibit 3. Note also that the vertical axis of average constituent capitalization is in logarithmic scale (and that dispersion and capitalization data represent 10-year averages). We suppress the segment name if the calculation is based on the underlying stock-level data (hence “S&P 500” instead of “S&P 500 stocks”).

**Exhibit 4: An Inverse Relationship Between Constituent Size and Dispersion**

Source: S&P Dow Jones Indices LLC. Data based on annualized monthly dispersion and capitalization figures from October 2008 to September 2018. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance. Past performance is no guarantee of future results.

Generally speaking, the negative relationship plotted in Exhibit 4 demonstrates a broader truth: **higher dispersion frequently comes with a trade-off against greater granularity in constituents.**

**MEASURING RELATIVE RESEARCH VALUE: CAPACITY-ADJUSTED DISPERSION**

If we suppose that active positions are scaled in proportion to available capitalization, then all else being equal, **recommendations pertaining to a market segment with double the dispersion are just as valuable as equally accurate recommendations pertaining to a market segment with double the capitalization.**

In order to facilitate direct comparisons, we define a benchmark’s **capacity-adjusted dispersion (CAD)** as the product of the dispersion of the index’s constituents and their average size.

\[
CAD = \text{Dispersion} \times \text{Average Constituent Size}
\]

In Exhibit 4, the farther up and to the right that an index plots, the greater its capacity-adjusted dispersion.
The concept of capacity-adjusted dispersion has intuitive appeal; it might be interpreted as the potential scale of the available bets (capitalization) multiplied by the potential magnitude of the returns to successful bets (dispersion).

This formulation allows us to examine a given benchmark at different degrees of granularity. Instead of analyzing constituent stocks, we can group those stocks into segments such as sectors or countries. Of course, if we measure the dispersion of single-country returns within a global benchmark, we also need to measure the average capitalization at the single-country level to compute the capacity-adjusted dispersion.

Capacity-adjusted dispersion also has a particular advantage—it is quite simple to observe empirically. Benchmark weights, capitalizations, and constituent returns are all that is required in order to calculate the capacity-adjusted dispersion in each period. Exhibit 5 shows the evolution of the capacity-adjusted dispersion for the S&P 500.

Exhibit 5 shows how the increased concentration of capital into the largest names of the S&P 500 during the late 1990s technology bubble...

...accentuated the degree of active opportunity.

Exhibit 5: Monthly Capacity-Adjusted Dispersion of the S&P 500

Source: S&P Dow Jones Indices LLC. Data based on annualized monthly dispersion and capitalization figures from January 1990 and September 2018. Chart is provided for illustrative purposes. Past performance is no guarantee of future results.

It is illustrative to compare Exhibit 5 with simple S&P 500 dispersion as illustrated in Exhibit 2. While the peak in dispersion during the technology bubble was already clear from Exhibit 2, Exhibit 5 shows how the increased concentration of capital into the largest names of the S&P 500 during the late 1990s accentuated the degree of active opportunity. More recently, the increases in concentration and
capitalization observed in the S&P 500 during 2018 have produced a striking increase in capacity-adjusted dispersion, at a time when dispersion itself has increased only moderately. This implies an increased absolute capacity for overweighting or underweighting active positions and a greater opportunity for active outperformance (or embarrassment) in absolute terms.

**RELATIVE MEASURES OF CAPACITY-ADJUSTED DISPERSION**

While absolute measures are useful, it seems more natural to express the value of recommendations in relative terms. For example, what value might one place on advice to buy or sell certain S&P 500 sectors in comparison to S&P 500 stocks?

Taking the capacity-adjusted dispersion of S&P 500 stocks as the fundamental unit of comparison, Exhibit 6 displays the ratio between the 10-year average capacity-adjusted dispersion of various markets compared to that of S&P 500 stocks.

**Exhibit 6: Capacity-Adjusted Dispersion Across Various Markets**

<table>
<thead>
<tr>
<th>Market Representation</th>
<th>Relative Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P 500 Sectors</td>
<td>7.49</td>
</tr>
<tr>
<td>S&amp;P Developed Ex-U.S. BMI Countries</td>
<td>6.79</td>
</tr>
<tr>
<td>S&amp;P Developed Ex-U.S. BMI Sectors</td>
<td>6.6</td>
</tr>
<tr>
<td>S&amp;P Emerging BMI Sectors</td>
<td>3.07</td>
</tr>
<tr>
<td>S&amp;P Emerging BMI Countries</td>
<td>1.92</td>
</tr>
<tr>
<td>S&amp;P 500 Information Technology</td>
<td>1.58</td>
</tr>
<tr>
<td>S&amp;P 500 Energy</td>
<td>1.21</td>
</tr>
<tr>
<td>S&amp;P 500 Communication Services</td>
<td>0.73</td>
</tr>
<tr>
<td>S&amp;P 500 Health Care</td>
<td>0.72</td>
</tr>
<tr>
<td>S&amp;P 500 Financials</td>
<td>0.66</td>
</tr>
<tr>
<td>S&amp;P 500 Consumer Discretionary</td>
<td>0.64</td>
</tr>
<tr>
<td>S&amp;P 500 Consumer Staples</td>
<td>0.6</td>
</tr>
<tr>
<td>S&amp;P Europe 350</td>
<td>0.55</td>
</tr>
<tr>
<td>S&amp;P 500 Industrials</td>
<td>0.49</td>
</tr>
<tr>
<td>S&amp;P Developed Ex-U.S. BMI</td>
<td>0.43</td>
</tr>
<tr>
<td>S&amp;P/TOPIX 150</td>
<td>0.3</td>
</tr>
<tr>
<td>S&amp;P 500 Materials</td>
<td>0.27</td>
</tr>
<tr>
<td>S&amp;P Emerging BMI</td>
<td>0.23</td>
</tr>
<tr>
<td>S&amp;P 500 Real Estate</td>
<td>0.16</td>
</tr>
<tr>
<td>S&amp;P 500 Utilities</td>
<td>0.11</td>
</tr>
<tr>
<td>S&amp;P MidCap 400</td>
<td>0.05</td>
</tr>
<tr>
<td>S&amp;P Frontier BMI</td>
<td>0.03</td>
</tr>
<tr>
<td>S&amp;P SmallCap 600</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Source: S&P Dow Jones Indices LLC. Data based on annualized monthly dispersion and capitalization figures from October 2008 to September 2018. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance. Past performance is no guarantee of future results.
Accordingly, Exhibit 6 offers a quantitative basis for direct comparisons. We might say, for example, that the most valuable research to a manager benchmarked against the S&P Developed Ex-U.S. BMI would be to know which countries to overweight or underweight, while research with a similar IC on S&P 500 sectors could be worth more than seven times as much as research on S&P 500 stocks.

The idea that research at the sector or country level is more valuable, followed by research on stock selection, is a powerful finding and supports a top-down approach to asset allocation. This has significant implications for the asset management industry, with potential for less reliance on sell-side analysts’ coverage of individual stocks.

Additionally, Exhibit 6 allows us to determine the relative difference in IC required to make the value of research in one market equivalent to the value of research in another. For example, the IC of reports covering S&P Europe 350 stocks would have needed to be 1.8 times larger, on average, than that covering S&P 500 stocks to make research coverage on the former as valuable as those of the latter, ceteris paribus.

In passing, we note that the scale of differences in Exhibit 6 allows for the possibility that research conducted in more granular segments such as small-cap or frontier markets must have considerably higher predictive power to be attractive. This offers some theoretical support to the widespread (and somewhat intuitive) assumption that research conducted at a more granular level, and on less-popular segments of the market, might offer the potential for a higher success rate.

**CONCLUSION**

The most fundamental aspect in valuing research is the accuracy (or predictive power) of the quantitative or qualitative signals supplied. However, while the information content of research reports might be estimable ex-post, it is hard to know in advance. Investors and intermediaries considering the cost of receiving research must instead form a view on the likely performance of the associated predictions. Once they have done so, the notion of capacity-adjusted dispersion may assist in deciding where the rewards for outperformance, and the capacity to capture them, combine to offer the best market or segments for the expression of active positions.
APPENDIX A: A STYLISTIC EXAMPLE OF DISPERSION’S ROLE IN ACTIVE RETURNS

In order to construct a simplified and stylistic example,\(^{18}\) suppose for a single time period there is an independently and identically distributed collection of zero-mean random excess returns \(S_i\) with standard deviation \(D\). In notational terms:

\[
S_i \text{ are single period excess constituent returns for } i = 1, 2, ...
\]

with expectation and volatility \(E(S_i) = 0\) and \(\sigma(S_i) = D\), respectively.

Suppose we are provided with near-random digital buy or sell recommendations \((R_i = \pm 1)\), equal to plus or minus one, that display a correlation \((IC)\) to the excess return of the constituents, but are otherwise noisy.

\[
R_i = 1 \text{ or } R_i = -1 \text{ are buy or sell recommendations for } i = 1, 2, ...; \text{ and}
\]

\[
IC = Corr(R_i, S_i) \text{ is the correlation between signal and ex-post returns.}
\]

Suppose also that recommendations to buy or sell are equally likely, and that the (unweighted) returns from taking recommendations were independent.

\[
E(R_i) = 0; \text{ and}
\]

\[
Corr(R_iS_i, R_jS_j) = 0 \text{ for any different pair of distinct recommendations } i \neq j.
\]

Completing the setup, suppose the active weights are—in absolute terms—determined in advance according to some “capacity,” and there are exogenous active weights \(w_i\) such that:

\[
w_i = \text{active weight in the } i^{th} \text{ recommended security with associated active position equal to } w_iR_i.
\]

We also have the total active share \((TAS)\) and effective number of active positions \((M)\),

\[
TAS = \sum w_i \text{ and } M = \left(\frac{1}{\sum w_i^2}\right).
\]

Note that if the \(R_i\) are a series of 1s and -1s with equal probability of either, there is then only one possible distribution overall for the \(R_i\): binomial with unit variance.

Then, in our simplification, the expected excess return \(E(P)\) of the active portfolio is given by:

\[
E(P) = E\left(\sum w_iR_iS_i\right) = E(R_iS_i) \times \sum w_i
\]

Recalling that \(Corr(X, Y)\sigma(X)\sigma(Y) = Cov(X, Y) = E(XY) - E(X)E(Y)\), we may substitute \(R_i\) for \(X\) and \(S_i\) for \(Y\), along with our earlier definitions and conditions to obtain:

The Value of Research: Skill, Capacity, and Opportunity

October 2018

\[ E(R_iS_i) = IC \times D \]

And so

\[ E(P) = IC \times TAS \times D. \]

Meanwhile, and recalling the definition of \( M \), the variance \( Var(P) \) of the excess returns of the active portfolio is given by:

\[ Var(P) = Var \left( \sum w_i R_i S_i \right) = Var(R_i S_i) \times \sum w_i^2 = \left[ \frac{E(R_i^2 S_i^2) - E(R_i S_i)^2}{M} \right] \]

and since \( R_i^2 = 1 \),

\[ Var(P) = \frac{D^2(1 - IC^2)}{M}. \]

With the variance and return thus described, the ratio of excess return to excess return volatility, is equal to:

\[ IR = IC \times \sqrt{M} \times TAS \times \frac{1}{\sqrt{1 - IC^2}} \]

If the \( IC \) is not large (say 0.1 at most) then dividing by \( \sqrt{1 - IC^2} \) will have only a small order effect (at worst altering an information ratio below 1 in the third decimal place), which completes the derivation.

**APPENDIX B: INDEX VERSUS EQUAL-WEIGHTED AVERAGES**

As defined earlier, the capacity-adjusted dispersion for a weighted collection of market segments is equal to their dispersion times their average size. The calculation of the two components, dispersion and size, is as follows.

Dispersion is index weighted in the sense that for a single period and without any annualization,

\[ Dispersion = \sqrt{\sum w_i (C_i - P)^2} \]

Where \( w_i \) is the benchmark weight of the \( i^{th} \) constituent at the start of the period, \( C_i \) is the return of the \( i^{th} \) constituent over the period, and \( P \) is the benchmark return, equal to the weighted sum of constituent returns.

The average size of constituents is also a benchmark-weighted average,

\[ Average Size = \sum w_i F_i \]

Where \( F_i \) is the free-float market capitalization of the \( i^{th} \) constituent (taken at the start of the period). If the benchmark is capitalization weighted, and the total benchmark capitalization is represented by \( TM \), then each \( F_i \) is equal to \( w_i TM \) and we may write
The Value of Research: Skill, Capacity, and Opportunity

October 2018

**Average Size** = \( \sum w_i^2 TM \)

If \( M = (1/\sum w_i^2) \) is the effective number of constituents in the market benchmark, then it is immediately clear that we can rewrite the average size as the total benchmark capitalization divided by the effective number of constituents.

The notion of the effective number of constituents has intuitive applications. Consider the simple example of a market with five stocks as shown in Exhibit B1.

<table>
<thead>
<tr>
<th>STOCK</th>
<th>CAP WEIGHT (%)</th>
<th>EQUAL WEIGHT (%)</th>
<th>CAP WEIGHT SQUARED (%)</th>
<th>EQUAL WEIGHT SQUARED (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>90.0</td>
<td>20.0</td>
<td>81.0</td>
<td>4.0</td>
</tr>
<tr>
<td>B</td>
<td>2.5</td>
<td>20.0</td>
<td>0.1</td>
<td>4.0</td>
</tr>
<tr>
<td>C</td>
<td>2.5</td>
<td>20.0</td>
<td>0.1</td>
<td>4.0</td>
</tr>
<tr>
<td>D</td>
<td>2.5</td>
<td>20.0</td>
<td>0.1</td>
<td>4.0</td>
</tr>
<tr>
<td>E</td>
<td>2.5</td>
<td>20.0</td>
<td>0.1</td>
<td>4.0</td>
</tr>
</tbody>
</table>

**SUM OF SQUARES (%)**  81.3  20.0

\( 1/\text{SUM OF SQUARES} \)  1.23  5.00

Source: S&P Dow Jones Indices LLC. Table is provided for illustrative purposes.

Stock A is gargantuan compared to stocks B-E and so dominates a cap-weighted index of the market. If we square the capitalization weights, add them up, and take the inverse, we can compute the effective number of positions of 1.23. The intuition behind the 1.23 is that although there are five names in the universe, four of them don’t matter much; effectively we have 1.23 stocks. (On the other hand, if we assume that the index is equally weighted, then the effective number of stocks is the same as the actual number.)

As noted above, the capacity in our computation of capacity-adjusted dispersion is the index’s total capitalization divided by the effective number of constituents. However, the computation of dispersion also relies on capitalization weighting. It’s legitimate to wonder if we have capitalization-weighted once too often (that is, twice instead of once).

The capitalization weighting in the dispersion calculation reflects the fact that most indices are themselves capitalization weighted. If a manager takes active positions in proportion to capitalization, his excess rewards or losses versus the benchmark (in percentage terms) should have a capitalization weighting applied to their distribution.

Moreover, in most practical circumstances, and particularly for broader-based indices, if managers are faced with a per-security cost of research, they are likely to pay for and implement a more limited set of overweights and underweights and—given the higher capacity they can take in response—are likely to be biased toward recommendations in constituents with higher benchmark weights.

---

19 The effective number is the reciprocal of the Herfindahl-Hirschman Index concentration measure, and both statistics have become part of the literature of diversification in financial theory. The two definitions appear to have been developed independently; the latter in the familiar context of portfolio concentration via Albert Hirschman’s *The Paternity of an Index* (The American Economic Review, Vol. 54, No. 5, September 1964, p. 176); the former as a measure of the effective number of political parties with varying numbers of parliamentary seats (Laakso, M. and R. Taagepera, “The ‘Effective’ Number of Parties: A Measure with Application to West Europe,” Comparative Political Studies, 12:1, April 1979).
Our analysis imposes several stylistic assumptions on the potential manner in which managers might overweight or underweight securities in general. Implicit in the manner in which we calculated the capacity-adjusted dispersion, we assume that managers are more likely to 1) make active bets in larger constituents than smaller constituents, and 2) take larger absolute active positions in larger constituents. Further research might be conducted to see if either of these two assumptions are confirmed in the empirical data of actual active positions held by active portfolios, but as indicated in this appendix, different types of active behavior might be modeled simply through making variations in the weights used to compute average size and cross-sectional dispersion.

In order to demonstrate the potential impact of the choice between the effective and actual number of constituents, Exhibit B2 offers an alternative version of Exhibit 6, supposing capacity were computed via an equal-weight average of constituent sizes.

**Exhibit B2: Capacity-Adjusted Dispersion Under Equal-Weight Average Size Calculations**

![Diagram showing capacity-adjusted dispersion under equal-weight average size calculations.](source)

Source: S&P Dow Jones Indices LLC. Data based on annualized monthly dispersion and capitalization figures from October 2008 to September 2018. Chart is provided for illustrative purposes and reflects hypothetical historical performance. Please see the Performance Disclosure at the end of this document for more information regarding the inherent limitations associated with back-tested performance. Past performance is no guarantee of future results.

The significant difference between the value for sectors and countries in the S&P Developed Ex-U.S. BMI displayed in Exhibit B2 contrasts to their relative similarity in Exhibit 6 and illustrates the impact of adjusting for concentration or not. As of September 2018, there were 23 countries represented in the index, eight of which with an index weight of less than 1%—and an effective number of countries equal to nine.
<table>
<thead>
<tr>
<th>S&amp;P DJI RESEARCH CONTRIBUTORS</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunjiv Mainie, CFA, CQF</td>
<td>Global Head</td>
<td><a href="mailto:sunjiv.mainie@spglobal.com">sunjiv.mainie@spglobal.com</a></td>
</tr>
<tr>
<td>Jake Vukelic</td>
<td>Business Manager</td>
<td><a href="mailto:jake.vukelic@spglobal.com">jake.vukelic@spglobal.com</a></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GLOBAL RESEARCH &amp; DESIGN</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AMERICAS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aye M. Soe, CFA</td>
<td>Americas Head</td>
<td><a href="mailto:aye.soe@spglobal.com">aye.soe@spglobal.com</a></td>
</tr>
<tr>
<td>Phillip Brzenk, CFA</td>
<td>Director</td>
<td><a href="mailto:phillip.brzenk@spglobal.com">phillip.brzenk@spglobal.com</a></td>
</tr>
<tr>
<td>Smita Chirputkar</td>
<td>Director</td>
<td><a href="mailto:smita.chirputkar@spglobal.com">smita.chirputkar@spglobal.com</a></td>
</tr>
<tr>
<td>Rachel Du</td>
<td>Senior Analyst</td>
<td><a href="mailto:rachel.du@spglobal.com">rachel.du@spglobal.com</a></td>
</tr>
<tr>
<td>Bill Hao</td>
<td>Director</td>
<td><a href="mailto:wenli.hao@spglobal.com">wenli.hao@spglobal.com</a></td>
</tr>
<tr>
<td>Qing Li</td>
<td>Director</td>
<td><a href="mailto:qing.li@spglobal.com">qing.li@spglobal.com</a></td>
</tr>
<tr>
<td>Berlinda Liu, CFA</td>
<td>Director</td>
<td><a href="mailto:berlinda.liu@spglobal.com">berlinda.liu@spglobal.com</a></td>
</tr>
<tr>
<td>Hamish Preston</td>
<td>Associate Director</td>
<td><a href="mailto:hamish.preston@spglobal.com">hamish.preston@spglobal.com</a></td>
</tr>
<tr>
<td>Maria Sanchez</td>
<td>Associate Director</td>
<td><a href="mailto:maria.sanchez@spglobal.com">maria.sanchez@spglobal.com</a></td>
</tr>
<tr>
<td>Kelly Tang, CFA</td>
<td>Director</td>
<td><a href="mailto:kelly.tang@spglobal.com">kelly.tang@spglobal.com</a></td>
</tr>
<tr>
<td>Hong Xie, CFA</td>
<td>Director</td>
<td><a href="mailto:hong.xie@spglobal.com">hong.xie@spglobal.com</a></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>APAC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Priscilla Luk</td>
<td>APAC Head</td>
<td><a href="mailto:priscilla.luk@spglobal.com">priscilla.luk@spglobal.com</a></td>
</tr>
<tr>
<td>Arpit Gupta</td>
<td>Senior Analyst</td>
<td><a href="mailto:arpit.gupta1@spglobal.com">arpit.gupta1@spglobal.com</a></td>
</tr>
<tr>
<td>Akash Jain</td>
<td>Associate Director</td>
<td><a href="mailto:akash.jain@spglobal.com">akash.jain@spglobal.com</a></td>
</tr>
<tr>
<td>Liyu Zeng, CFA</td>
<td>Director</td>
<td><a href="mailto:liyu.zeng@spglobal.com">liyu.zeng@spglobal.com</a></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EMEA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sunjiv Mainie, CFA, CQF</td>
<td>EMEA Head</td>
<td><a href="mailto:sunjiv.mainie@spglobal.com">sunjiv.mainie@spglobal.com</a></td>
</tr>
<tr>
<td>Leonardo Cabrer, PhD</td>
<td>Senior Analyst</td>
<td><a href="mailto:leonardo.cabrer@spglobal.com">leonardo.cabrer@spglobal.com</a></td>
</tr>
<tr>
<td>Andrew Cairns</td>
<td>Senior Analyst</td>
<td><a href="mailto:andrew.cairns@spglobal.com">andrew.cairns@spglobal.com</a></td>
</tr>
<tr>
<td>Andrew Innes</td>
<td>Associate Director</td>
<td><a href="mailto:andrew.innes@spglobal.com">andrew.innes@spglobal.com</a></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>INDEX INVESTMENT STRATEGY</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Craig J. Lazzara, CFA</td>
<td>Global Head</td>
<td><a href="mailto:craig.lazzara@spglobal.com">craig.lazzara@spglobal.com</a></td>
</tr>
<tr>
<td>Chris Bennett, CFA</td>
<td>Director</td>
<td><a href="mailto:chris.bennett@spglobal.com">chris.bennett@spglobal.com</a></td>
</tr>
<tr>
<td>Fei Mei Chan</td>
<td>Director</td>
<td><a href="mailto:feimei.chan@spglobal.com">feimei.chan@spglobal.com</a></td>
</tr>
<tr>
<td>Tim Edwards, PhD</td>
<td>Managing Director</td>
<td><a href="mailto:tim.edwards@spglobal.com">tim.edwards@spglobal.com</a></td>
</tr>
<tr>
<td>Anu R. Ganti, CFA</td>
<td>Director</td>
<td><a href="mailto:anu.ganti@spglobal.com">anu.ganti@spglobal.com</a></td>
</tr>
<tr>
<td>Howard Silverblatt</td>
<td>Senior Index Analyst</td>
<td><a href="mailto:howard.silverblatt@spglobal.com">howard.silverblatt@spglobal.com</a></td>
</tr>
</tbody>
</table>
PERFORMANCE DISCLOSURE

The S&P 500 Real Estate was launched September 19, 2016. All information presented prior to an index’s Launch Date is hypothetical (backtested), not actual performance. The back-test calculations are based on the same methodology that was in effect on the index Launch Date. However, when creating back-tested history for periods of market anomalies or other periods that do not reflect the general current market environment, index methodology rules may be relaxed to capture a large enough universe of securities to simulate the target market the index is designed to measure or strategy the index is designed to capture. For example, market capitalization and liquidity thresholds may be reduced. Complete index methodology details are available at www.spdji.com. Past performance of the Index is not an indication of future results. Prospective application of the methodology used to construct the Index may not result in performance commensurate with the back-test returns shown.

S&P Dow Jones Indices defines various dates to assist our clients in providing transparency. The First Value Date is the first day for which there is a calculated value (either live or back-tested) for a given index. The Base Date is the date at which the Index is set at a fixed value for calculation purposes. The Launch Date designates the date upon which the values of an index are first considered live: index values provided for any date or time period prior to the index’s Launch Date are considered back-tested. S&P Dow Jones Indices defines the Launch Date as the date by which the values of an index are known to have been released to the public, for example via the company’s public website or its datafeed to external parties. For Dow Jones-branded indices introduced prior to May 31, 2013, the Launch Date (which prior to May 31, 2013, was termed “Date of introduction”) is set at a date upon which no further changes were permitted to be made to the index methodology, but that may have been prior to the Index’s public release date.

The back-test period does not necessarily correspond to the entire available history of the Index. Please refer to the methodology paper for the Index, available at www.spdji.com for more details about the index, including the manner in which it is rebalanced, the timing of such rebalancing, criteria for additions and deletions, as well as all index calculations.

Another limitation of using back-tested information is that the back-tested calculation is generally prepared with the benefit of hindsight. Back-tested information reflects the application of the index methodology and selection of index constituents in hindsight. No hypothetical record can completely account for the impact of financial risk in actual trading. For example, there are numerous factors related to the equities, fixed income, or commodities markets in general which cannot be, and have not been accounted for in the preparation of the index information set forth, all of which can affect actual performance.

The Index returns shown do not represent the results of actual trading of investable assets/securities. S&P Dow Jones Indices LLC maintains the Index and calculates the Index levels and performance shown or discussed, but does not manage actual assets. Index returns do not reflect payment of any sales charges or fees an investor may pay to purchase the securities underlying the Index or investment funds that are intended to track the performance of the Index. The imposition of these fees and charges would cause actual and back-tested performance of the securities/fund to be lower than the Index performance shown. As a simple example, if an index returned 10% on a US $100,000 investment for a 12-month period (or US $10,000) and an actual asset-based fee of 1.5% was imposed at the end of the period on the investment plus accrued interest (or US $1,650), the net return would be 8.35% (or US $8,350) for the year. Over a three year period, an annual 1.5% fee taken at year end with an assumed 10% return per year would result in a cumulative gross return of 33.10%, a total fee of US $5,375, and a cumulative net return of 27.2% (or US $27,200).
GENERAL DISCLAIMER

Copyright © 2018 S&P Dow Jones Indices LLC. All rights reserved. STANDARD & POOR’S, S&P, S&P 500, S&P 500 LOW VOLATILITY INDEX, S&P 100, S&P COMPOSITE 1500, S&P MIDCAP 400, S&P SMALLCAP 600, S&P GIVI, GLOBAL TITANS, DIVIDEND ARISTOCRATS, S&P TARGET DATE INDICES, GICS, SPIVA, SPDR and INDEXOLOGY are registered trademarks of Standard & Poor’s Financial Services LLC, a division of S&P Global (“S&P”). DOW JONES, DJ, DJIA and DOW JONES INDUSTRIAL AVERAGE are registered trademarks of Dow Jones Trademark Holdings LLC (“Dow Jones”). These trademarks together with others have been licensed to S&P Dow Jones Indices LLC. Redistribution or reproduction in whole or in part are prohibited without written permission of S&P Dow Jones Indices LLC. This document does not constitute an offer of services in jurisdictions where S&P Dow Jones Indices LLC, S&P, Dow Jones or their respective affiliates (collectively “S&P Dow Jones Indices”) do not have the necessary licenses. Except for certain custom index calculation services, all information provided by S&P Dow Jones Indices is impersonal and not tailored to the needs of any person, entity or group of persons. S&P Dow Jones Indices receives compensation in connection with licensing its indices to third parties and providing custom calculation services. Past performance of an index is not an indication or guarantee of future results.

It is not possible to invest directly in an index. Exposure to an asset class represented by an index may be available through investable instruments based on that index. S&P Dow Jones Indices does not sponsor, endorse, sell, promote or manage any investment fund or other investment vehicle that is offered by third parties and that seeks to provide an investment return based on the performance of any index. S&P Dow Jones Indices makes no assurance that investment products based on the index will accurately track index performance or provide positive investment returns. S&P Dow Jones Indices LLC is not an investment advisor, and S&P Dow Jones Indices makes no representation regarding the advisability of investing in any such investment fund or other investment vehicle. A decision to invest in any such investment fund or other investment vehicle should not be made in reliance on any of the statements set forth in this document. Prospective investors are advised to make an investment in any such fund or other vehicle only after carefully considering the risks associated with investing in such funds, as detailed in an offering memorandum or similar document that is prepared by or on behalf of the issuer of the investment fund or other investment product or vehicle. S&P Dow Jones Indices LLC is not a tax advisor. A tax advisor should be consulted to evaluate the impact of any tax-exempt securities on portfolios and the tax consequences of making any particular investment decision. Inclusion of a security within an index is not a recommendation by S&P Dow Jones Indices to buy, sell, or hold such security, nor is it considered to be investment advice.

These materials have been prepared solely for informational purposes based upon information generally available to the public and from sources believed to be reliable. No content contained in these materials (including index data, ratings, credit-related analyses and data, research, valuations, model, software or other application or output therefrom) or any part thereof ("Content") may be modified, reverse-engineered, reproduced or distributed in any form or by any means, or stored in a database or retrieval system, without the prior written permission of S&P Dow Jones Indices. The Content shall not be used for any unlawful or unauthorized purposes. S&P Dow Jones Indices and its third-party data providers and licensors (collectively “S&P Dow Jones Indices Parties”) do not guarantee the accuracy, completeness, timeliness or availability of the Content. S&P Dow Jones Indices Parties are not responsible for any errors or omissions, regardless of the cause, for the results obtained from the use of the Content. THE CONTENT IS PROVIDED ON AN “AS IS” BASIS. S&P DOW JONES INDICES PARTIES DISCLAIM ANY AND ALL EXPRESS OR IMPLIED WARRANTIES, INCLUDING, BUT NOT LIMITED TO, ANY WARRANTIES OF MERCHANTABILITY OR FITNESS FOR A PARTICULAR PURPOSE OR USE, FREEDOM FROM BUGS, SOFTWARE ERRORS OR DEFECTS, THAT THE CONTENT’S FUNCTIONING WILL BE UNINTERRUPTED OR THAT THE CONTENT WILL OPERATE WITH ANY SOFTWARE OR HARDWARE CONFIGURATION. In no event shall S&P Dow Jones Indices Parties be liable to any party for any direct, indirect, incidental, exemplary, compensatory, punitive, special or consequential damages, costs, expenses, legal fees, or losses (including, without limitation, lost income or lost profits and opportunity costs) in connection with any use of the Content even if advised of the possibility of such damages.

S&P Global keeps certain activities of its various divisions and business units separate from each other in order to preserve the independence and objectivity of their respective activities. As a result, certain divisions and business units of S&P Global may have information that is not available to other business units. S&P Global has established policies and procedures to maintain the confidentiality of certain non-public information received in connection with each analytical process.

In addition, S&P Dow Jones Indices provides a wide range of services to, or relating to, many organizations, including issuers of securities, investment advisers, broker-dealers, investment banks, other financial institutions and financial intermediaries, and accordingly may receive fees or other economic benefits from those organizations, including organizations whose securities or services they may recommend, rate, include in model portfolios, evaluate or otherwise address.